Pedagogical Strategies Using Van Hiele's Levels of Geometric Thinking in Learning Geometry Among Malaysian Year Five Students

Pui Feng Ser
Faculty of Education, University of Malaya, Malaysia
reneeser@siswa.um.edu.my

Mohd Shahril Nizam Bin Shaharom
Faculty of Education, University of Malaya, Malaysia
nizamsaril@um.edu.my

Mohd Razip Bin Bajuri
Faculty of Education, University of Malaya, Malaysia
razip.bajuri@um.edu.my

Choon Tak Chan
Faculty of Education, University of Malaya, Malaysia
choontak@siswa.um.edu.my

Abstract

Geometry deals with the study of different shapes in plane or solid. Although geometry is taught as one of the important areas in the Malaysian Mathematics curriculum, research studies indicate that many Malaysian students' geometric thinking is not at the level they are expected to be and they experienced learning difficulties similar to those students encountered throughout the world. This study was conducted to determine the levels of students' geometric thinking among forty-five participants from a class of mixed-ability year five students. Adapted Wu's Geometry Test was used to review the levels of students' geometric thinking. Data was presented in the forms of mean and percentage using graphs and tables. The results indicated that the student participants' levels of geometric thinking were low. The students' level of geometric thinking was found at Level L1-Visualization. Findings retrieved from the study shows that this research is very helpful in identifying primary school student's levels of geometric thinking based on van Hiele Model and appropriate pedagogical strategies are proposed to promote their geometric thinking.

Keywords: geometry, van Hiele Model, levels of geometric thinking, year five students, Malaysia

INTRODUCTION

In line with 21st century learning trends, The Malaysia Education Blueprint 2013- 2025 reformed eleven operational shifts (MOE, 2013). Since 2011, the implementation of a curriculum changes at primary school level began in phases with year one students. The new curriculum was named as National Primary School Standard-Based Curriculum (*Kurikulum Standard Sekolah Rendah, KSSR*), aimed to produce Malaysian citizens who are knowledgeable and competent that can function in the 21st century. Hence this KSSR curriculum has been fully implemented in 2016 with year six students.

In the Malaysian KSSR Mathematics syllabus, from year one to year two, the students are able to distinguish, describe, and classify geometric shapes and solids in their daily life. From year three to year four, the students are able to understand the geometric shapes through the symmetry, parallel and perpendicular lines, and angles as well. They learn to calculate perimeter and area of geometric shapes and volume of solids. From year five to year six, the students are able to use the geometric properties to solve geometry problems involving angles in polygons, perimeter and area of composite geometric shapes and volume of composite solids (MOE, 2020).

Geometry deals with the study of different shapes in plane or solid. Since geometry is one of the basic skills to be mastered by the students since they are at the primary level in Malaysia, the students are introduced to the geometry at the visual level in the first three years; in the fourth to sixth year, they are encouraged to progress to the descriptive level and calculate the perimeter and area of 2D shapes and volume of 3D solids (MOE, 2020).

In recent years, Malaysian education has aimed to move away from rote learning and memorization toward providing more challenging and deeper thinking (MOE, 2020). Although geometry is taught as one of the important areas in the mathematics curriculum, research studies indicate that many Malaysian students' geometric thinking is not at the level they are expected to be (Chew & Noraini, 2012; Md Yunus et al., 2019; Tan et al., 2015). Many students at primary level experienced learning difficulties similar to those students encountered throughout the world (Tan et al., 2015).

Background of the Study

Many studies (Al-ebous, 2016; Andini et al., 2018; Chua et al., 2017; Ismail & Rahman, 2017) globally have shown that van Hiele model examines levels of geometric thinking among students. The model explains how students learn geometry hierarchically (Ismail & Rahman, 2017). Other studies (Pasani, 2019; Tan et al., 2015) that employ geometry learning based on the van Hiele theory find that fifth grade students have difficulties in understanding the characteristics of the 2D shapes; that learning using van Hiele's levels can solve students' difficulties in understanding geometric shapes; that primary school students still have difficulties in identifying the 2D shapes; that students' geometric comprehension positively correlates to the achievement in their study of geometric shapes; and that students' geometric performance was improved when using the van Hiele theory.

In an experimental study by Pasani (2019) on 150 primary year five students of the medium and low performances, the researchers claimed that students' level of geometric thinking in van Hiele-based geometry learning is higher than that of students in conventional learning. This finding is congruent with van Hiele's theory that comprehension of geometric concepts is a skill developed through a process that builds from the previous schemes, previous understanding, and network of relationship among the concepts (Pasani, 2019).

van Hiele Model is a model designed by Pierre van Hiele and his wife Dina van Hiele (1984/1956) which consists of three domains, namely levels of geometric thinking, characteristics of the geometry acquisition and phase-based instruction (Al-ebous, 2016). As reported by Crowley (1987), the original van Hiele model is a developmental model that identifies five levels of thought processes through which students learn geometry.

The first level is visualization. At this level, the student can recognize geometric shapes. The student gets to know and identify certain geometric shapes based on the entity of the whole object and not on its components or traits. The second level in van Hiele's model is analysis. In this level, the student can identify with certain geometric traits and concepts. The third level is informal deduction. At this level, the student can see the relation between shapes and then the student creates that relation. They can relate existing knowledge and form arguments to come with correct generalization. The fourth level is deduction. In this level, the student understands the meaning and importance of deduction and the role of postulates, theorems and proofs. They are able to create proof based on their own understanding. The fifth level is rigor. At this level, the student understands how to work in an axiomatic system. They are able to form a more abstract deduction (Crowley, 1987).

The van Hiele model enables the researchers and the teachers to explain why many students encounter difficulties in their geometry lessons. The model also offers the teachers to apply and practice in order to promote their students' levels of geometric thinking (Fuys et al., 1988; van Hiele, 1986). It originally posited five sequential and hierarchical discrete levels of geometric thinking (Senk, 1989; Usiskin, 1982). Two different numbering schemes commonly used in the literature are as follows: Level 0 (L0) through to Level 4 (L4) and Level 1 (L1) through to Level 5 (L5) (Senk, 1989). The original numbering scheme used by van Hiele was Level 0 (L0) through to Level 4 (L4) which refers to visualization, analysis, informal deduction, deduction and rigor (van Hiele, 1986). In this study, the researchers used Level 1 to Level 5 to allow the students who are not able progress to any level to be defined as Level 0.

The van Hiele levels of geometric thinking based on five levels of reasoning that are sequential and hierarchical, as well as progressing from the concrete to the more abstract is shown in Table 1.

Table 1: ***Description of the van Hiele Geometric Thinking Levels***

Level	Description
1 Visualization	Visual perception is present. A figure is seen as a total entity and as a particular shape. Properties are not considered in the identification of a shape.
2 Analysis	The figure is now identified by its geometric properties rather than by its appearance. However, the properties are seen in isolation. Therefore, definitions among essential information and not yet attained.
3 Informal Deduction	The importance of properties is now seen. The properties are not treated in isolation. Instead, the properties are ordered logically and relationships existing among the properties are recognized.
4 Formal Deduction	Logical reasoning ability is developed. Geometric proofs are created with meaning. Necessary and sufficient conditions are used with strong conceptual understanding.
5 Rigor	Theorems in different postulation systems are established and analyzed.

Source: Mistretta (2000)

van Hiele (1986) believes that in order to proceed from one level of thinking to another one it is necessary to follow the five stages of teaching, namely: (i) inquiry: students are introduced to the area of geometric figures; (ii) directed orientation: the tasks are presented in such a way that students gradually learn particular properties; (iii) explanation: a teacher connects the lessons learned with the correct terms; (iv) free orientation: a teacher presents tasks that can be completed in different ways and enables children to become more proficient with what they already know; (v) integration: students are given the opportunity to connect their acquired knowledge. The activities are summarized which allows students to connect the existing and new knowledge.

In phase 1- inquiry, students develop vocabulary and concepts for a particular task. In phase 2- directed orientation, students actively engage in teacher-directed tasks. In phase 3- explication, students are given the opportunity to verbalize their understanding. In phase 4- free orientation, students are challenged with discovering their ownways of completing each task. In phase 5- integration, students summarize what they have learned from the lesson (Mistretta, 2000).

To assist teachers with incorporating the concepts of the model into the geometry classroom, Mistretta (2000) provided an outline of these five phases as illustrated in Table 2. These five phases of learning indicate how students can progress through the levels of learning and the manner in which the model is integrated into the lesson planning.

Table 2 *Five phases of geometry learning*

Phase	Description
1-Inquiry	Discussions are held where the teacher learns of the students' prior knowledge and experience with the subject matter at hand.
2-Directed Orientation	The teacher provides activities that allow students to become more acquainted with the material being taught.
3-Explication	A transition between reliance on the teacher and students' self-reliance is made.
4-Free Orientation	The teacher is attentive to the inventive ability of the students. Tasks that can be approached in numerous ways are presented to the students.
5-Integration	The students summarize what they learned during the lesson.

Source: Mistretta (2000)

As viewed by Mistretta (2000), phase 1 involves both the teacher and the students. Discussions are held in which the teacher learns of the students' prior knowledge and experience with the subject matter at hand, while the student becomes familiar with the new material. During phase 2, the teacher provides activities which allow students to become more acquainted with the current material being taught. The students are operating empirically during this phase. The third phase is a transition between reliance on the teacher and reliance on one's self. The students learn to express their opinions about the structures observed during class discussions. The teacher, at this point, needs to assist the students with proper vocabulary. In the fourth instructional phase, the teacher is attentive to the inventive ability of the students. Tasks that can be approached in numerous ways should be presented to the students during this phase. During the fifth phase, the students must summarize what was learned during the period of class instruction, although nothing new is introduced at this phase. The students are considered as having progressed to a higher level of thinking in the van Hiele model at the end of this phase.

Hence, learning activities for the topics of geometry are indeed encouraged to be built and arranged based on van Hiele’s phases of learning geometry (Choi-Koh, 2000; Halat & Peker, 2008). There were even a few past studies conducted that revealed that the activities arranged based on van Hiele’s phases of learning geometry had given positive impacts to students in various aspects including the students’ understanding and their levels of geometric thinking (Choi-Koh, 2000; Chew & Noraini, 2012).

Problem Statement

The results of International Trends in Mathematics and Science Study (TIMSS) in 2019 showed a fall in Malaysia’s rank in Mathematics compared to results in 2015. Results showed that the performance of the students in the geometry field was 461 lower than the scores 465 in 2015 (TIMSS, 2019). Average Malaysian Students' Mathematics scores in The International Trends in Mathematics and Science Study (TIMSS) showed inconsistency from 1999 to 2019. The score was decrease from 519 to 440 in 2011. The achievement was increase to 465 in 2015 and slightly decrease to 461 in 2019 as shown in Figure 1.

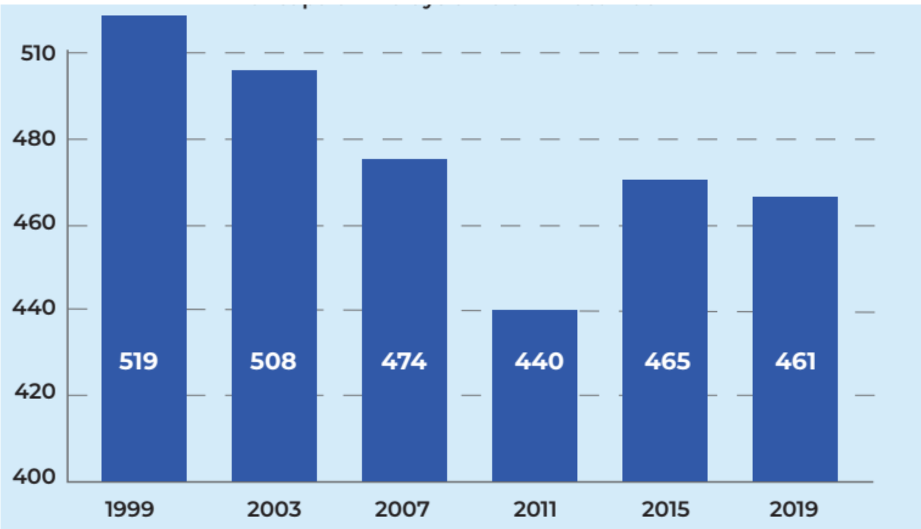

Figure 1: Average Malaysian Students' Mathematics Score in TIMSS from 1999 to 2019

Source: TIMSS Report 2019

According to PISA 2018 report, average Malaysian students' scores were lower than the Organization for Economic Cooperation and Development (OECD) in reading, mathematics and science. Some 59% of students in Malaysia attained Level 2 or higher in mathematics compared to OECD average at 76%. However, there are 2% of Malaysian students who scored at Level 5 or higher in mathematics compared to OECD average at 11%. Meanwhile, six Asian countries with outstanding performance are China (44%), Singapore (37%), Hong Kong (China) (29%), Macao (China) (28%), Chinese Taipei (23%) and Korea (21%).Students in these countries were found that can perform mathematical model for complex situations. They can also select, compare and evaluate appropriate problem-solving strategies, and geometry is one of the domains that being assessed in the program.

In a geometry classroom, students have difficulties in solving geometry problems without the experience in discovering geometry relationships (Andini et al., 2018). Therefore the researchers carried out a qualitative study with 30 grade six students in Indonesia using van Hiele geometry test named VHG developed by Usiskin in 1982. The results indicated that the students' ability in identifying geometrical properties is very low and only 35% of the students reach at informal deduction which is Level 3 as they are unable to use the geometrical models to solve the problems. In spite of using technology software of GeoGebra, the year two students are unable to analyse 3D solids and visualize 2D shapes as well as 3D solids (Ismail & Rahman, 2017).

The failure by many students to understand geometry generated debates in many countries (Andini et al., 2018; Chua et al., 2017; Md Yunus et al., 2019; Fujita et al., 2017). According to NCTM (2000), year five students should have mastered Level 1 (visualization) in geometry and be progressing into Level 2 (analysis). Nevertheless, there is not much information on geometry attainment of the students based on van Hiele levels in Malaysia (Md Yunus et al., 2019). The researchers investigated three different teaching methods on 96 year five students in a public school and found that learning geometry using van Hiele Theory had successfully eliminated the students who were operating at lower levels at the onset of the experiment and they had progressed to at least Level 1 (visualization) after the interventions.

The contemporary curriculum does not recommend the appropriate teaching and learning approaches for the teachers to promote higher levels of geometric thinking among the primary school students (Md Yunus et al., 2019; Musdi & Gsunita, 2018). In Malaysia, the prevailing aims of learning geometry are naming, drawing, using geometric symbols and recognizing geometrical shapes and solids (MOE, 2020). Most of the learning activities are much more emphasized in arithmetic in a prescribed way, often lacking any meaningful challenge (Chew & Noraini, 2012). The lesson plans and worksheets used in teaching and learning do not help the students to develop the geometry understanding but merely contain materials, formulas and examples of problem solving (Musdi & Gsunita, 2018).

It has been reported that the teaching and learning of mathematics in Malaysia has been too teacher-centred and students are not given enough opportunities to develop their own thinking (Abdul Halim & Effendi, 2013; Md Yunus et al., 2019). Therefore, learning difficulties encountered in geometry may be attributed by inadequate experience provided to students. Students encountered difficulties in understanding geometry concepts as the conventional method of learning, less student-centered and the topics learnt were not related to students' daily lives. As a result, students become lazy and weary during the teaching and learning process can affect their learning outcomes and the heavy use of formulas and drilling exercises made the students less trained with the creativity and understanding of geometry concepts (Siregar et al., 2019).

Purpose of Study

This study aims to determine the levels of geometric thinking among year five students in a Chinese public primary school in Malaysia. Prior research and much attention have been drawn focusing on the academic performance among the primary school students. Nevertheless, there is a lack of study on the geometric thinking among the primary school students because most studies are done in other countries. Thus, it is crucial to determine the levels of geometric thinking levels among the primary students and appropriate pedagogical strategies are proposed to promote their geometric thinking.

Research Objectives

Regarding the purpose of this study, the researchers aim to achieve the following research objectives:

1. To determine the levels of geometric thinking among year five students based on van Hiele Model; and
2. To propose the appropriate pedagogical strategies to promote students' geometric thinking based on van Hiele Model.

Research Questions

With corresponding to the objectives of this study, the research questions sound as below:

1. What are the levels of geometric thinking among year five students based on van Hiele Model?
2. What are the appropriate pedagogical strategies can be proposed to promote students' geometric thinking based on van Hiele Model?

Methodology

This study was carried out as a preliminary study by involving 45 year five students. There are 23 boys and 21 girls. The samples consisted of an intact mixed ability year five class from a Chinese public school in the district of Hulu Langat in Selangor. This research focus on the year five mathematics topics of geometry for the following reasons: First, geometry is taught as an important area in the mathematics curriculum in Malaysia; Second, research studies (Chew & Noraini, 2012; Md Yunus et al., 2019; Tan et al., 2015) showed that many students' geometric thinking skill are not at the level they are expected to. Third, geometry is taught from primary school level to secondary school level, the students need to acquire knowledge of geometry, however previous studies (Abdul Halim & Effendi, 2013; Chew & Noraini, 2012; Md Yunus et al., 2019) indicated that students always have difficulties in understanding geometry.

The data was collected using a geometry test of Adapted Wu's Geometry Test. There are 75 multiple choices questions given to the students for the first three levels of geometric thinking based on van Hiele model. Each level of van Hiele geometric thinking consists of 25 questions. At the first level-visualization is the level to identify the geometric figures. The test material of this level is recognizing the 2D shapes and 3D solids. The students use simple language and do not identify the properties of geometric figures. At the second level -analysis is the level to determine various properties of the 2D shapes and 3D solids while questions for the third level-informal deduction is to develop the understanding of relations among the 2D shapes and 3D solids (Tan, 2016).

In Adapted Wu's Geometry Test, there are five basic concepts of geometry that has been focused which are triangles, squares, rectangles, cubes and cuboids according to year five Malaysian syllabus of mathematics (Tan, 2016). The students develop their geometric thinking based on general properties and relations among these five basic concepts of geometry (Md Yunus et al., 2019). Students are considered to have achieved one level if 3 or more out of 5 questions are answered correctly. Therefore, using the same criteria set by Usiskin (1982), the passing rate of this test was set at 60%.

This study only focuses on the first three levels of van Hiele model, therefore the discussion will be focused on van Hiele's geometric thinking of visualization, analysis, and informal deduction. Descriptive statistics was used to find the mean and percentage distribution of the data. Collected data will measure every student based on the van Hiele levels of geometric thinking.

To gain the validity of the test, it was presented to a university lecturer and two primary school mathematics teachers. The original Adapted Wu's Geometry Test was refined to align with the contemporary Mathematics syllabus and proper terminologies that were used in the school context. The test was translated into Chinese language version to ease the students to understand the questions. Therefore, the final test consisted of 75 multiple choices items and each question carried four marks. To check the test's reliability, the internal consistency was calculated using Kuder-Richardson and the reliability coefficient value was 70.5% which showed a high degree of reliability.

Findings

Based on the graphic on Table 3 and Figure 2, van Hiele's geometric thinking levels among year five students are inclined from the first level to low at the third level. The means of the first three levels are shown in Table 3. Data was analysed from results of Adapted Wu's Geometry Test among 45 year five students are 63.64% at first level (visualization), 45.42% at second level (analysis), and at third level (informal deduction), the mean is 38.4%.

Table 3: Mean of Level 1-3

Level	Mean	SD
L1	63.64	15.674
L2	45.42	11.573
L3	38.4	9.396

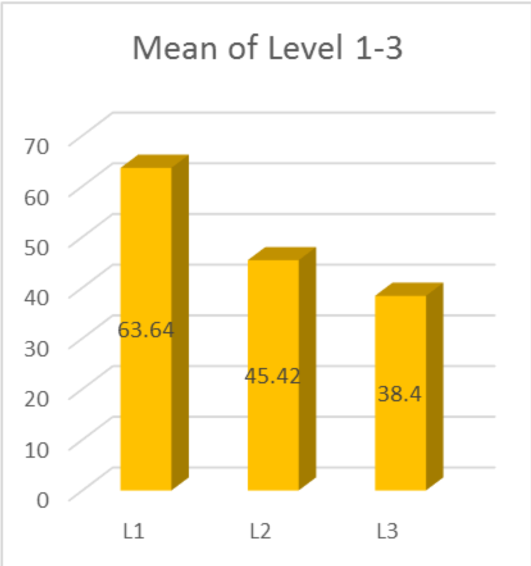

Figure 2: Mean of each level among students

Supported by the Table 3 and Figure 2, the results show that the students perform the best in the first level. Most of the students answer the questions correctly as they can identify the appearance of the geometric shapes and solids. The students score 45.42% at level 2 because they are unable to determine the properties of the shapes and solids. At level 3, the students are weak in making relations among the shapes and solids, the mean is only 38.4%.

Table 4: Passing Rate of Level 1-3

Level/Geometry Shapes and Solids	L1	L2	L3
Triangle	86.7	26.7	11.1
Square	73.3	55.6	15.6
Cube	82.2	55.6	53.3
Rectangle	84.4	20	17.8
Cuboid	77.8	40	42.2

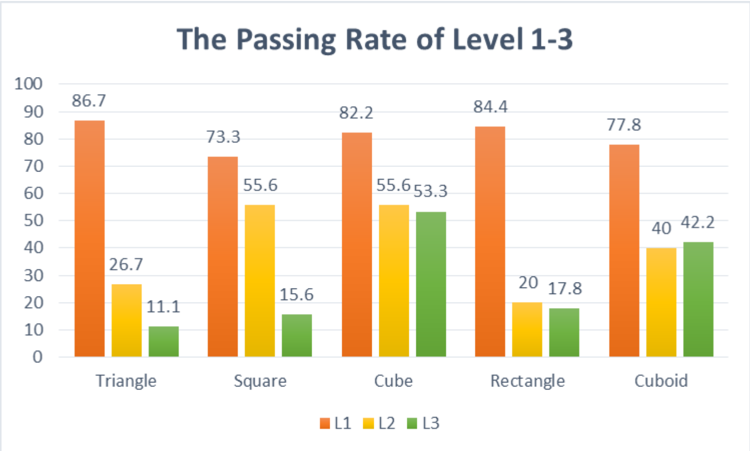

Figure 3: The passing rate of each level among students

Table 4 and Figure 3 show precisely the students' geometric thinking according to each level in the topics of triangles, squares, rectangles, cubes and cuboids. The majority of the students can recognize the geometric shapes and solids visually. At the first level test, 86.7% of the students pass in the topic of triangle, 84.4% of the students pass in the topic of rectangle, 82.2% pass with the topic of cube, 77.8% pass with the topic of cuboid and 73.3% pass in the topic of square. At the second level test, 55.6% of the students pass in the topic of square and cube respectively, 40% of them pass in the topic of cuboid, 26.7% pass with the topic of triangle and the least 20% pass in the topic of rectangle. At the third level, 53.3% of the students pass in the topic of cube, 42.2% pass in the topic of cuboid, 17.8% pass with the topic of rectangle, only 15.6% pass in the topic of square and the least 11.1% pass in the topic of triangle.

The data indicated that students use their daily life experience to discover the appearance of the geometry shape and solids visually. Therefore, they can score high at level 1 of geometric thinking. However, minority of the students can achieve level 2 and 3 because most students' ability in determining geometrical properties and relations among 2D shapes and 3D solids is still very low.

Studies conducted by several researchers have also revealed phenomena which are similar (Chew & Noraini, 2012; Fatih & Ilham, 2018; Md Yunus et al., 2019; Tan et al., 2015). Their findings showed that the students in their studies only operated at the lower level of van Hiele's levels of geometric thinking. More attention is needed to promote the students in acquiring informal deduction (Level 3) in particular 3D geometry (Ismail & Rahman, 2017).

Discussions

Many researchers (Argaswari, 2018; Maja & Tatjana, 2015; Musdi & Gsunita, 2018) used activities and instruction based on van Hiele theory to assist students in promoting their levels of understanding of basic geometry concepts and subsequent improvement in achievement. Musdi and Gsunita (2018) developed mathematical devices using van Hiele Model in teaching grade 8 students. The learning devices in their study included lesson plans for the teachers and worksheets for the students. The model was proved to improve students' geometric understanding because the students learn geometry step by step. Argaswari (2018) found that the model is useful to develop a teaching and learning module for grade 7 students in quadrilateral topics.

A set of teaching activities based on van Hiele phase-based model was developed by Maja and Tatjana (2015) to accelerate the transition of students to a higher level of geometric thinking. The effectiveness of the activities was established students at certain tasks, at the removal of squares and rectangles of different quadrilaterals did not progress following their exposure to teaching. The researchers proposed that greater emphasis should be put on teaching of various quadrilaterals, possibly also on their properties and relationship.

Fujita et al. (2017) proposed a small group discussion followed by the whole class discussion to allow the students to share their ideas about their learning. The problem was first introduced to enable the students to share their ideas when investigating the topic. Thus, a correct answer was then verified by refuting the answers using various learning resources (Fujita et al., 2017). Students are collaborating in groups and conducting research and learning from each other enhances students' communication skills and team work (Pasani, 2019).

Difficulties in learning geometry is caused by the misconceptions experienced by the students so the teachers should plan suitable learning activities to help the students to build their knowledge constructively (Ismail et al., 2020). Students also experienced difficulties at analysis level because they are unable to interpret problems into mathematical model therefore the teachers should assist the students to develop their problem-solving ability (Sulistiowati et al., 2018). Al-ebous (2016) suggested that a positive educational environment should be created to encourage the students to take part in discussion and search for information themselves without giving the ready answers and the activities in the teacher guide book should be planned based on the van Hiele Model.

Conclusion

In this study, a preliminary analysis was carried out to identify year five students' van Hiele's levels of geometric thinking before the intervention. According to the collected data analysis, this study found that most of the year five students reach the first level of van Hiele geometric thinking at the mean of 63.64%. This finding was in line with the results of research conducted by Ismail and Rahman (2017) and Andini et al. (2018) that the primary school students' geometric thinking needs improvement using appropriate teaching and learning activities based on van Hiele model.

In conclusion, Malaysia aims to achieve above the global average and be in the top one-third of countries participating in PISA and TIMSS by 2025, in line with the Malaysia Education Blueprint 2013-2025 (MOE, 2013). Geometry concepts help students to relate geometric structures with other concepts of mathematics they learnt and solve problems that they encounter in the real world through relationships they build (Fatih & Ilham, 2018). Thus, geometric thinking should be promoted through the integration of the school subjects. When a teacher creates a communicative learning environment for the students, the students tend to share ideas to reach the learning objective, make better sense of applying the geometry concepts in their daily lives and make their learning meaningful.

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