Profit Share Labour Arrangements in Competitive Markets

Evangelos Koutronas\textsuperscript{a}, Siew-Yong Yew\textsuperscript{b}, Kim-Leng Goh\textsuperscript{c}, Mohd Zulfadhli Zakaria\textsuperscript{d}

Abstract: This paper explores the concept of profit sharing as a prospective source of labour income. The paper suggests a paradigm shift: a synthesis of differing perspectives in wage formation to redefine the concept of labour income. The proposed arrangements attempt to conceptualise the productive capacity of human capital. The rearrangement of labour income disassociates the basic wage from the wage premium. Furthermore, it keeps the labour-cost- to-total-cost ratio constant: the basic wage is categorised as a fixed cost, whereas the wage premium is categorised as a variable cost. Finally, some comparative statics results are derived.

Keywords: profit sharing, perfect competition, labour economics, human capital

JEL Classification: D41, J01, J31, J33

Article received: 4 October 2016; Article accepted: 11 May 2017

1. Introduction

Alternative incentive-based mechanisms are increasingly becoming popular to induce employee motivation. Employers are willing to offer above the market-clearing compensation and benefits if the marginal product of labour does not equal the reservation wage. Nevertheless, reward policies are subject to boundary conditions of employee participation and incentive comparability. Arguably, variable compensation prescribes a necessary but not sufficient condition for higher effort, given current labour arrangements. Recent developments have identified market differentials that affect the traditional underlying factors of production and change the perception in

\textsuperscript{a} Social Security Research Centre, Faculty of Economics and Administration, University of Malaya, 50603 Kuala Lumpur, Malaysia. Email: evangel_gr@um.edu.my

\textsuperscript{b} Department of Economics, Faculty of Economics and Administration, University of Malaya, 50603 Kuala Lumpur, Malaysia. Email: yewsy@um.edu.my

\textsuperscript{c} Department of Applied Statistics, Faculty of Economics and Administration, University of Malaya, 50603 Kuala Lumpur, Malaysia. Email: klgoh@um.edu.my

\textsuperscript{d} Social Security Research Centre, Faculty of Economics and Administration, University of Malaya, 50603 Kuala Lumpur, Malaysia. Email: mohdzulfadhli@um.edu.my
corporate performance. The new trajectory of business environment presupposes a paradigm shift in wage formation. The paradigm should be based on the desire to reshape income distribution patterns, rather than to cut them back.

Literature on efficiency wage explain real wage differentials; nonetheless, the argument that the effort-wage elasticity is equal to unity seems theoretically implausible. Additional labour costs either in the form of fixed employment costs (Schmidt-Sorensen, 1990), labour taxes (Pisauro, 1991), turnover costs (Lin & Lai, 1994), or job matching costs (Jellal & Zenou, 1999), clearly indicate that the effort-wage elasticity is less than unity.

This paper explores the concept of profit sharing as a prospective source of labour income. It suggests a paradigm shift: a synthesis of differing perspectives in wage formation to redefine the concept of labour income. The proposed labour income arrangements attempt to conceptualise the productive capacity of human capital. The rearrangement of labour income disassociates the basic wage from the wage premium.

Most studies are limited to explaining the capital-utilization decisions of labour-managed business models (Betancourt & Clague, 1977; Jensen & Meckling, 1979; Meade, 1972; Vanek, 1970; Weitzman, 1983, 1985). The current empirical research is concerned with the micro-level benefits of profit sharing, focusing mainly on the impact of profit-sharing on business performance rather than on the actual estimation of profit-sharing.

The paper is organised as follows: Section two presents theoretical and empirical considerations on employee effort and compensation. Section three sets the context of the efficiency wage. Section four introduces the concept of redistribution profits. In a Neo-classical setting, section four examines profit share wage in a profit maximizing firm. In section five, the consequences of the introduction of profit sharing are discussed. Sections six and seven examine employee individual effort and the optimal wage, respectively. The final section concludes the paper. The Appendix contains figures and proofs.

2. Literature Review

The need for self-actualisation through recognition, achievement, and personal growth are intrinsic characteristics of human behaviour (Herzberg, Mausner, & Snyderman, 1959; Maslow, 1943). Salaries are necessary to get the job done, but not the sole motivation. An employee put his or her best effort in the job even if the latter is unrelated to his or her qualifications in terms of job field or position. Nonetheless, this employee will be partially or entirely disinterested in performing his or her duties if meaningful activities
are not involved. Lack of motivation usually leads to underperformance and absenteeism thus described as employee passive behaviour in the workplace with employees just meeting only the minimum performance requirements.

On the other hand, heavy workload can also be discouraging. In both cases, the employee will start looking for another more suitable position that commensurate with his qualifications and experience; thus, it increases the firm’s turnover rate (Koslowsky & Krausz, 2002). Alternative reward mechanisms include commissions, bonuses and in-kind incentives to stimulate employee performance. However, actual compensation is unrelated to performance in many instances (Jensen & Murphy, 1990). There is a negative correlation between earned income and the recognition of employee's higher standard of performance.

In the field of organisational behaviour, reinforcement theory (Skinner, 1938; Skinner & Ferster, 1957), expectancy theory (Vroom, 1964), equity theory (Adams, 1963; Lawler, 1968), and agency theory (Arrow, 1972; Wilson, 1968) have been used to justify compensation policies to increase performance. Based on Thorndike's Law of Effect (1911), reinforcement theory indicates that high employee performance accompanied with monetary compensation will be repeated in the future. In addition, compensation objectives boost wage rates to an efficient level required to recruit, motivate or retain the best employees giving the employer a comparative advantage.

Expectancy theory goes one step further arguing that individual’s behaviour or action is incentive determined. In essence, motivation mechanisms determine individual behavior selection among other factors by the desirability of outcome. According to equity theory, employees’ performance is determined by how fair they perceive their employers to be in assessing their performance and the rewards scheme, and if their return-performance ratio is comparable (with outsiders). In agency theory terms, the use of various compensation performance mechanisms seeks to align the interests of the employer with those of the employees. It is a result of asymmetric information: the employer cannot directly ensure that employees are always acting in the company’s best interest since the employer fails to verify the performance of employees due to lack of monitoring or ineffective incentives. The employer should clearly define intended objectives, reward policies and disbursement methods. Any misconception about the fairness of the compensation programme will engender employee distrust and discouragement inhibiting in the end firm effectiveness (Levinthal, 1988).

Weitzman (1987) argues that firms always have needs to proceed with new employment arrangements. Profit-sharing offers firm flexibility. Wages do not display a downward rigidity, as the Keynesian theory dictates, but it creates favourable market conditions where employee displacements and voluntary terminations are significantly reduced.
In fact, several studies provide empirical evidence about the positive impact of profit-sharing on firm performance (Freeman, 2008; Hashimoto, 1975, 1979; Huselid, 1995); on firm productivity (Blasi, Freeman, Mackin, & Kruse, 2008; Bryson & Freeman, 2008; Carstensen, Gerlach, & Hubler, 1995; Doucouliagos, 1995; FitzRoy & Kraft, 1987; Kraft & Ugarkovic, 2005; Kruse, 1992; OECD, 1995); on labour productivity (Bental & Demougin, 2006; Conyon & Freeman, 2004; Weitzman & Kruse, 1990); on distributing efficiency risk (Brouwer, 2005); on boosting employment performance resulting a decrease on the marginal cost of labour (Jerger & Michaelis, 1999; Weitzman, 1985), and; on investing in human capital (Azfar & Danningier, 2001; Gielen, 2007; Parent, 2004). Additionally, remuneration packages allow companies greater flexibility in managing wage costs without actually affecting the worker’s income (Weitzman, 1983, 1985). Based on Weitzman’s theory, Pohjola (1987) and Anderson and Devereux (1989) showed that collective bargaining over both fixed and variable labour income would lead to more job creation. If the marginal cost of labour equals to labour rate, then the firm can achieve full employment resulting in a higher compensation rate.

Although profit-sharing practices have been widely adopted in developed countries (Gerhart & Milkovich, 1992; Heneman & Schwab, 1979; Kim, 1988; Milkovich & Newman, 1993; Noe, Hollenbeck, Gerhart, & Wright, 1994; Pendleton, Poutsma, Ommersen, & Brewster, 2001; Poutsma, 2003; Weitzman, 1987, 1995), the productivity effect is ambiguous. Jensen and Meckling (1979), and Kruse (1992) demonstrated negative correlation between productivity and profit-sharing. They concluded that profit-sharing may be subject to all the difficulties and problems associated with the horizon, the common-property, non-transferability, control and entry problem just like in the Yugoslav-type model. Homlund (1990) and Nickell and Layard (1999) showed that there is a positive correlation between profit-sharing and employment in the short run, which gradually diminishes in the long run – shifting upward towards the prevailing wage. Trade unions though are sceptical regarding the profit-sharing concept. Profit-sharing results in higher total labour income, where in turn, trade unions will demand the prevailing wage to increase. However, hiring outsiders will be at the expense of insiders because the profit per worker declines. Sinn (1999) suggested that if only insiders receive profit shares, then their total income will improve. This argument may hold as long as the number of outsiders is low.

Koskela and Konig (2011) argued that profit-sharing may fail due to “free rider” problem. Some of the employees may shirk their responsibilities, if profits are equally distributed because it gives fewer incentives to employees to be productive. Equal distribution of profits was the main characteristic of the Yugoslavian labour-managed model where the stream of the firm’s net earnings was equally distributed to employees. Those enterprises operated
under the ultimate control of those who work in it. Unlike conventional firms, employees were entitled to property rights and decision-making power exogenously, given by the political, social, and legal systems (Putterman, 2008). An individual’s dual nature (owner and employee) was subject to conflict of interest creating distortions in decision-making process: decisions were taken with gnomon their personal interest rather than the best interests of the company they served posing serious threats to the functioning of those firms (Prasnikar & Prasnika, 1986).

Profit-sharing is purely individual, performance-related plan that provides cash and/or in-kind rewards to employees depending on the company's profitability. It can take the form of group-based compensation equally distributed to group members. In reality, a broad variety of compensation practices, including merit pay, cash bonuses, goal achievement rewards, and various gain-sharing plans have been successfully implemented in large-cap and small-cap firms, labour-intensive and capital-intensive industries, manufacturing and services, and industries with balanced and seasonal profits. Profit-sharing can be a valuable component in human resources management strategy. Compensation options are usually viewed more suitable for small firms where management can present an objective and accurate analysis of an employee’s performance. A compensation programme should be tailored according to organisational and employee needs.

The existence of an additional source of income in the formal labour market suggests differential changes in the current wage formation. The proposed labour income arrangements attempt to quantify the productive capacity of human capital. The rearrangement of the labour income disassociates the statutory/prevailing wage, whichever is higher, from the wage premium. Furthermore, it keeps the labour-cost-to-total-cost ratio constant: the statutory/prevailing wage is categorised as a fixed cost, whereas the wage premium is categorised as a variable cost.

3. Setting the Context

Efficiency wage theory postulates that effort and real wage move in tandem. Solow (1979) argued that wage is the only determinant of effort: \( e = e(w) \), \( e'(w) > 0, e(0) = 0 \). Yellen (1984) argued that firm’s output is contingent upon its labour force, \( L \), and their effort, \( e \):

\[
Q = F(e(w)L), \quad F'(\cdot) > 0, \quad F''(\cdot) < 0
\]

Akerlof and Yellen (1984) formulated below profit maximization based on Solow’s findings:
\[ \max_{w,L} \pi [e(w)L] = F[e(w)L] - wL \]  

The first order conditions in respect to \( w \), and \( L \) derive

\[ F'[e(w^*)L] e'(w^*) = 1 \]  

(3)

\[ F'[e(w^*)L] e(w^*) = w^* \]  

(4)

Solve equation (4) for \( F'[e(w)L] \) and substitute (4) into (3) yields

\[ w^* \frac{e'(w^*)}{e(w^*)} = 1 \]  

(5)

Equation (5) indicates effort-wage equilibrium. The unitary elasticity considers any marginal change in wage irrelevant. Production is expressed in terms of effective labour, \( e(w^*)L \). The firm aims to minimise the cost per efficiency unit of labour \( \frac{w^*}{e(w^*)} \), given the market-clearing wage. The market-clearing wage \( w^* \) equals the efficiency wage. The wage-effort relationship is presented in Figure 1.

**Figure 1:** Wage–effort relationship

![](image)


The efficiency wage is expected to be *Pareto* optimal only if the equilibrium wage is sufficient to compensate employees for their effort. According to Fair Wage Hypothesis (Akerlof, Rose, & Yellen, 1988; Akerlof...
employer-employee relationship is a matter of perception: how employees perceive their level of effort, how their performance is rewarded, and how their remuneration is competitive to insiders and outsiders (Alchian & Demsetz, 1972; Koskela & Konig, 2011; Samuelson, 1977). This “subjective” perception of wage fairness leads to a fair-wage paradox (Moroy, 2013): firms expect employees to exert maximum effort whereas employees follow the “law of least effort”, minimizing the amount of effort they exert in order to obtain desirable outcomes (Ferrero, 1894; Kingsley Zipf, 1949; Poole, 1985). It is a plausible that a divergence between actual and fair wage will cause employees to respond with reduced effort (Adams, 1963; Homans, 1961):

\[ e = \min \left( \frac{w}{w^*}, 1 \right) = \begin{cases} \frac{w}{w^*} : w < w^* \\ 1 : w \geq w^* \end{cases} \]  

(6)

where \( w \) denotes market clearing wage and \( w^* \) the efficiency wage. Equation (6) explicates the presence of unemployment. Unemployment occurs when the efficiency wage exceeds the market-clearing wage (Kahneman, Knetsch, & Thaler, 1986). In the case of a higher market-clearing wage, overpayment does not increase effort (Walster, Walster, & Berscheid, 1977). In economic terms, equation (6) can be translated as

\[ e = B \left( \frac{w}{w^*} \right)^n \]  

(7)

where \( B \) is the distribution parameter, and \( n \) is the elasticity parameter. Intuitively, perceived effort equals to expected value of remuneration. The distribution parameter \( B \) reflects the within-firm compensation equality, while the elasticity parameter \( n \) responds to compensation fairness in regards to the industry. With \( w \) and \( w^* \) fixed, those employees who do not receive efficiency wage may change actual effort or their perceived effort (Akerlof & Yellen, 1990).

Efficiency wages identify the cognitive and physical effort required to perform general tasks from an overview perspective. Hence, they may lead to dysfunctional behavioural responses in a multi-task working environment (Baker, 1992; Holmstrom & Milgrom, 1991). Complex activities usually require the use of higher-level skills and competencies that translate into higher levels of effort (Campbell, 1990, 1994). Heavy workloads can be discouraging resulting in employees seeking greener pastures that commensurate with their qualifications and experience; thus, it increases firm’s turnover rate (Koslowsky & Krausz, 2002).
Remuneration links reward with performance that usually exceeds market-clearing wage and performance criteria. Compensation differentials act as a complement to the market clearing wage (Bhargava & Jenkinson, 1995; Hart & Hubler, 1991; Wadhwani & Wall, 1990) and as an add-on to the efficiency wage (Bradley & Estrin, 1992; FitzRoy & Kraft, 1992). Both cases set an equilibrium higher than the *Walrasian* wage rate, and thus, involuntary unemployment occurs.

### 3.1 The Concept of “Redistribution” Profits

The representative firm aims to maintain the convergence to a stationary equilibrium or a *Pareto* improving path. It should be expected that with the relatively small number of economic agents (consumers and firms), the emergence of the new market equilibrium corresponds with a non-clearing price level, thereby it cannot be a *Walrasian* equilibrium. If a non-optimal equilibrium exists, then it cannot also be a *Pareto* optimal. This "subjective" perception of demand may pass through the revised market equilibrium, even if this is arbitrary beyond the initial market equilibrium (Benassy, 1975; Negishi, 1979). A feature of this framework is that it nests in a variety of alternative specifications for price setting, including the menu cost model (Akerlof & Yellen, 1985a, 1985b; Mankiw, 1985; Sheshinski & Weiss, 1977) the *Calvo* model (Calvo, 1983; King & Wolman, 1996) and *Boiteux-Ramsey* price equilibrium (Baumol, Panzar, & Willig, 1982; Boiteux, 1971; Braeutigam, 1979, 1984; Lipsey & Lancaster, 1956; Ramsey, 1927; Sadmo, 1975).

The menu cost model investigates the effect of marginal changes in prices on firm average costs. The price elasticity of demand determines whether the upstream cost is cost-absorbing or cost-amplifying, leading to nominal rigidity (also known as sticky prices). The *Calvo* model is built on the foundations of the menu cost model. The *Calvo* model incorporates a hazard function to examine the impact of idiosyncratic shocks to marginal costs due to the presence of inflationary imbalances. Finally, the *Boiteux-Ramsey* price equilibrium acknowledges the existence of quasi-optimal prices: the presence of inconsiderable costs is a necessary condition for price changes, but not a sufficient condition for quantity changes. The concept suggests the imposition of an optimal tax rule. A profit-maximising firm will choose *Boiteux-Ramsey* prices only if all markets are equally monopolistic or competitive. However, the model ignores the effect of government transfer payments or receipts on the final demand schedule (partial equilibrium) (Oum & Tretheway, 1988).

Following Hotelling (1938), Meade (1944) and Fleming (1944) on marginal cost pricing, the *Boiteux-Ramsey* model considers optimal conditions for infinitesimal deviations, given the same constraints. Taxation
does not ipso facto justify government transfers nor receipts. Groves (1948) introduced the concept of “tax neutrality”: taxation should have no role in the formulation of public policy. According to Kahn (1990), the term “tax neutrality” is perceived as a tax provision as well as a tax expenditure. Based on this notion, it is possible for a government intervention through regulation without the presence of public redistribution mechanisms, bypassing the role of a social planner. The imposition of welfare taxation rested on its alleged “neutrality” among the alternatives open to employees.

Price-cap regulation could possibly lead to a social optimum if the state specifies whether the allocation efficiency is met\(^2\) (Netz, 2000). Theoretical underpinnings of optimal taxation lie in the second fundamental theorem of welfare economics: any prescribed Pareto efficient allocation can be attained through a competitive equilibrium, given the market redistribution mechanisms. In a perfectly competitive setting, market equilibrium is determined to the point where price equals marginal cost for firm and product per se. Despite the fact that policy makers have emphasised "fairness" in pricing rather than economic efficiency, optimal taxation is generally distortive and limits the ability of laissez-faire approach (Auerbach & Hines Jr., 2001).

At first, lump sum taxation is considered to be Pareto efficient since it does not interfere with optimal market mechanisms. Optimal lump sum taxation is based on relevant economic individual characteristics, such as preferences, attributes and endowments. There is a broad consensus that this form of taxation is hardly feasible (Mirrlees, 1976), even if some of its forms are (Stern, 1982). On the other hand, the optimal commodity taxation should reflect the elasticity dynamics of supply and demand (Ramsey, 1927). However, commodity taxation seems problematic because it constrains social planners in achieving the best outcome for all parties involved by not considering all possible tax structures (Mankiw, Weinzierl, & Yagan, 2009).

Finally, the corporate income taxation theory suggests the qualitative distribution of the tax burden among the factors of production and output elasticities (Harberger, 1962). Corporate income taxation theory though faces two shortcomings: 1) policy changes are focused solely on the effects of personal income tax (Feldstein, 2008); and 2) tax legislation has narrowed the taxable base, declining substantially the effective corporate income tax rate (Fox & LeAnn, 2002).

Overall, the optimal tax policy has moved in directions suggested by empirical considerations with certain prominent features, even though they are not always definitive. The optimal tax theory posits that a tax system should be chosen to maximise social welfare subject to revenue constraints. The question, however, remains as to the extent to which the theoretical suggestions can be transformed into an operational framework where tax
design and tax reform take into account a country’s distinctive socioeconomic characteristics.

3.2 The Model

Following Yellen (1984), consider a perfect firm that produces output based on the number of employees, \( L \), it employs and on their effort, \( e \). Despite the fact that work core functions remain constant, employee effort varies with form, direction, intensity and duration of the motive which drives the behaviour (Geen, 1995; Heckhausen, 1991; Weiner, 1992). Based on Brehm’s (1989) theory of motivation, effort is associated with potential motivation and motivation intensity. Potential motivation describes the maximum physical and cognitive effort an individual would be willing to exert to effectively perform a series of general tasks, functions and responsibilities of a position. Motivation intensity corresponds with a collection of resources – all the talents, skills, innate abilities, intelligence, knowledge, education, training, experience, judgment, and wisdom – employees acquired individually and collectively that actually expand. Following the Variable Labour Effort Hypothesis (Mason, 1993), consider a perfect firm that adopts a pay-to-performance systems in an attempt to maximise employee effort:

\[
e = \max (w^*, \tilde{w}, 1)
\]  

(8)

where \( w^* \) denotes the efficiency wage and \( \tilde{w} \) the variable wage. Consistent with the efficiency wage theory, the model assumes that objective and subjective effort depends positively on the efficiency wage and variable wage, respectively. Consistent with Solow (1979), we assume that the base wage \( w^* \) and wage premium \( \tilde{w} \) are the only determinants of objective and subjective efforts, respectively. Objective effort refers to a prudent man’s rule: the reasonable physical or mental activity needed by an average person to exert in order to perform essential duties and responsibilities of a designated position. The subjective effort refers to job expectations: it captures the overall employee effort related to personal qualities and traits, previous experience and technical proficiency that enables the average employee to perform in an effective and efficient manner. All employees are assumed to have identical labour-productivity relationships in the form \( e = e(w) \), which consists of the objective effort \( \tilde{e} = \tilde{e}(w^*) \) and the subjective effort \( \hat{e} = \hat{e}(\tilde{w}) \), where

\[
\tilde{e}'(\cdot) > 0, \quad \tilde{e}'(0) = 0, \quad \hat{e}'(\cdot) > 0, \quad \hat{e}'(0) = 0
\]  

(9)

where \( \tilde{e} \) denotes the objective effort and \( \hat{e} \) the subjective effort. The disaggregation of individual effort between the objective and subjective
effort presupposes differential changes in the wage formation given the existing labour market equilibrium. The representative firm’s output takes the bivariate form

\[ Q = AF(\bar{\varepsilon}(w^*), \hat{\varepsilon}(\bar{\omega})L), \quad F_{w^*}'(\cdot, \cdot) > 0, \quad F_{\hat{\varepsilon}}''(\cdot, \cdot) < 0 \quad (10) \]

where \( A \) represents the total factor productivity, \( \bar{\varepsilon} \) the objective effort and \( \hat{\varepsilon} \) subjective effort. The average objective effort is defined as \( \bar{\varepsilon} = \frac{1}{L} \sum_{i=1}^{L} \bar{\varepsilon}_i \), and a marginal unit of effort per employee \( \frac{\partial \bar{\varepsilon}}{\partial \bar{\varepsilon}_i} = \frac{1}{L} \). We combine these inputs in a concave production function which is expressed as

\[ F[\bar{\varepsilon}(\bar{\omega})L, \hat{\varepsilon}(\bar{\omega})L] = \left[ \alpha(\bar{\varepsilon}(w^*)L)^{\frac{\varepsilon-1}{\varepsilon}} + \beta(\hat{\varepsilon}(\bar{\omega})L)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}, \]

\[ 0 < \alpha < 1, \quad 0 < \beta < 1 \quad (11) \]

The base wage \( w^* \) is the minimum amount of salary paid to perform regular job requirements. The distribution parameters \( \alpha \) and \( \beta \) reflect the labour intensity of effort and \( \varepsilon \), is the intra-temporal elasticity of substitution between basic and variable labour income. Like all standard CES functions, equation (3) yields the Cobb-Douglas specification as \( \varepsilon \to 1 \); a Leontief function with fixed factor proportions as \( \varepsilon \to 0 \); and a linear production function with perfect factor substitution when \( \varepsilon \to \infty \). The production function is homogeneous of degree one in its inputs, \( F(t\bar{\varepsilon}(w^*)L, \hat{\varepsilon}(\bar{\omega})L) \equiv F[\bar{\varepsilon}(w^*)L, \hat{\varepsilon}(\bar{\omega})L] \equiv tF[\bar{\varepsilon}(w^*)L, \hat{\varepsilon}(\bar{\omega})L], t > 0 \), and satisfies the properties of essentiality: \( F[\bar{\varepsilon}(0)L, \hat{\varepsilon}(0)L] = 0 \); constant returns to scale: \( F(c\bar{\varepsilon}(w^*)L, c\hat{\varepsilon}(\bar{\omega})L) = cF[\bar{\varepsilon}(w^*)L, \hat{\varepsilon}(\bar{\omega})L], c \geq 0 \); positive diminishing returns to inputs: \( \frac{\partial F}{\partial \bar{\varepsilon}} > 0, \frac{\partial^2 F}{\partial \bar{\varepsilon}^2} < 0, \frac{\partial F}{\partial \hat{\varepsilon}} > 0, \frac{\partial^2 F}{\partial \hat{\varepsilon}^2} < 0 \); and Inada conditions: \( \lim_{w^* \to 0} \left( \frac{\partial F}{\partial \bar{\varepsilon}} \right) = \lim_{\hat{\varepsilon} \to 0} \left( \frac{\partial F}{\partial \hat{\varepsilon}} \right) = \infty, \lim_{w^* \to \infty} \left( \frac{\partial F}{\partial \bar{\varepsilon}} \right) = \lim_{\hat{\varepsilon} \to \infty} \left( \frac{\partial F}{\partial \hat{\varepsilon}} \right) = 0 \). The price output \( P \) is indeterminate; it undermines the price-taking assumption leading the model to reach multiple equilibria. Therefore, we arbitrarily normalise prices: prices are measured in terms of a commodity bundle. In particular, price normalisation in terms of the first good, the arbitral constant equals to be \( \frac{1}{P_1} \) while normalized price of the first good becomes \( (P) \left( \frac{1}{P_1} \right) = 1 \). Numeraire price allows the model to demonstrate constant return to scale at any equilibrium stage by preserving the properties of homotheticity and linearity: \( F[\bar{\varepsilon}(w^*)L, \hat{\varepsilon}(\bar{\omega})L] = g(h(\bar{\varepsilon}(w^*)L, \hat{\varepsilon}(\bar{\omega})L)); F[\bar{\varepsilon}(w^*)L, \hat{\varepsilon}(\bar{\omega})L] = m\bar{\varepsilon}(\bar{\omega})L + n\hat{\varepsilon}(\bar{\omega})L + b. \)
The firm experiences regulatory welfare arrangements \( u, u \in \mathbb{R}^k \) imposed by the federal government as a percentage of its operating profits. This implies that the effective price differs from the market price. It is further assumed that prices reflect social costs, i.e. there are no externalities. The new price equilibrium relies on consumer surplus. Price is expressed as:

\[
\hat{P} = \{(\hat{p}, 1) | \hat{p} = (\hat{p}_1, \hat{p}_2, \ldots, \hat{p}_l) \gg 0\}
\]

(12)

where \( \hat{P} \) is the new consumer price bundle denoted by the new price and the initial price, which is a numeraire, \( (\hat{p}, 1) \), whereas the new producer normal price bundled is denoted by \( (\hat{\pi}, 1) \in \hat{P} \). Due to the rule of normalisation, only the output components of the two price bundles is being compared. The level of production for the competitive firm remains unchanged and retains the properties of non-emptiness, closedness, and free disposal. Let \( \hat{P} = (\hat{p}, u) \) and \( P = (p, u) \) denote, respectively, the effective price and the market price received by the firm given the welfare arrangements. The firm effective price corresponds to a percentage of the operating profits: \( \hat{P} = P(1 + u) \).

Operating profit is the profit earned from a firm's normal core business operations. This value does not include any profit earned from the firm's investments (such as earnings from firms in which the company has partial interest) and the effects of interest and taxes. In a perfect competitive market setting, the use of the accounting estimation of profit leads to the valuation of firm net cash flows, where the economics in this case are considered “redundant” (Beaver, 1981). In a monopolistic market setting, Fisher and McGowan (1983) argue that “…only to the extent that profits are indeed monopoly profits, accounting profits are in fact economic profits, and the accounting rate of return equals the economic rate of return” (pg. 82). The direct total wage cost for the firm will be the sum of the basic wage plus the operating profit-share:

\[
w^c = w^* + \xi \frac{\Pi^{op}}{L}
\]

(13)

where \( w^c \) refers to the total labour cost and \( \zeta, \xi \in \mathbb{R}^k \), corresponds to labour-cost-to-total-cost ratio. The basic wage is an initial wage paid to the employee, which usually represents the effective wage. The operating profit-share-wage on the other hand will be set by the firm. Following Schmidt-Sorensen (1990), consider a multi-period competitive firm that faces profit maximisation decision in continuous time expressed in real terms:
\[
\max_{w_t, \pi^o_t} \pi \left( \hat{\varepsilon}(w^*), L, \hat{\varepsilon} \left( \frac{\pi^o}{L} \right), L \right) = \int_{0}^{\infty} \left\{ (1 + u) A_t F_t - w_t' L_t - \xi \frac{\pi^o_t}{L_t} L_t \right\} e^{-rt} dt
\]

(14)

where \( R_t \equiv r_t \) corresponds to the real interest rate. Total factor productivity \( A_t \) grows at a constant rate \( g \), \( A_t = A_0 e^{gt} \), \( A_0 > 0 \), and labour input \( L_t \) changes over time according to \( L_t = L_0 e^{nt} \), \( L_0 > 0 \). The subscript parameter \( t \) stands for time and captures the effect of technical progress. The first order conditions in respect to \( w_t^*, \pi^o_t/L_t \) and \( L_t \) derive

\[
(1 + u) A_t \frac{1}{\varepsilon} F_t^{\varepsilon-1} L_t^{-\frac{1}{\varepsilon}} \hat{\varepsilon}_t'(w^*)^{-\frac{1}{\varepsilon}} = 1
\]

(15)

\[
(1 + u) A_t \frac{1}{\varepsilon} F_t^{\varepsilon-1} L_t^{-\frac{1}{\varepsilon}} \hat{\varepsilon}_t' \left( \frac{\pi^o}{L} \right)^{-\frac{1}{\varepsilon}} = \xi
\]

(16)

\[
(1 + u) A_t \frac{1}{\varepsilon} F_t^{\varepsilon-1} L_t^{-\frac{1}{\varepsilon}} = \frac{w_t^* + \xi \frac{\pi^o_t}{L_t}}{[\hat{\varepsilon}_t(w^*)]^{-\frac{1}{\varepsilon}} + \left[ \hat{\varepsilon}_t \left( \frac{\pi^o}{L} \right) \right]^{-\frac{1}{\varepsilon}}}
\]

(17)

The second order conditions imply that of the second derivative of the effort functions, \( \bar{\varepsilon} \) and \( \hat{\varepsilon} \) are negative: \(-\frac{1}{\varepsilon} \bar{\varepsilon}''(w^*) -\frac{(1+\varepsilon)}{\varepsilon}, -\frac{1}{\varepsilon} \hat{\varepsilon}'' \left( \frac{\pi^o}{L} \right) -\frac{(1+\varepsilon)}{\varepsilon} \). Substituting equation (17) into (16) and (15), the following can be expressed

\[
\eta_{\hat{e}w^*} \left( 1 + \frac{\xi \frac{\pi^o}{L}}{w^*} \right) = 1
\]

(18)

\[
\eta_{\hat{e} \pi^o/L} \left( 1 + \frac{w^*}{\xi \frac{\pi^o}{L}} \right) = \xi < 1
\]

(19)

where \( \eta_{\hat{e}w^*} \) and \( \eta_{\hat{e} \pi^o/L} \) denote the partial elasticities of objective and subjective efforts with respect to total labour income. Equations (18) and (19) show that profit sharing indirectly affects labour demand consistent with empirical findings (Anderson & Devereux, 1989; Cahuc & Dormont, 1997; Pohjola, 1987; Wadhwani & Wall, 1990). The aforementioned equations
show that the equilibrium effort – compensation exhibit an imperfect elastic behaviour since both partial elasticities are prominently lower than unity. Furthermore, the subjective effort elasticity is less than the objective effort elasticity. According to Euler’s Theorem, the sum of the elasticities of output with respect to factor inputs is equal to the degree of homogeneity of the production function:

$$\eta_{\theta} + \eta_{\pi^{op}} = 1$$  \hspace{1cm} (21)

Equation (21) identifies a causal intra- and interrelationship between compensation and effort. The division of effort results in interdependence between task contribution and individual performance that leads to reward interdependence. The implications of these findings suggest that effort complexity, introduced through the presence of high levels of interdependence, can intensify the positive performance effects of compensation and effort. The realistic modification of the profit function by incorporating profit sharing labour arrangements weakens the general notion of effort-wage elasticity of unity making Solow’s condition irrelevant.

4. Differentiability in the Profit Sharing Factor

The effects on real wage, variable wage and number of employees from a change in the profit sharing costs, $\pi^{op}_L$, are now outlined. At first, from equations (15) and (17) the Hessian matrix with respect to $\bar{w}^\varepsilon$ and L is shown before checking the impact of $\pi^{op}_L$:

$$\left[ (1 + u) A \frac{1}{\varepsilon} F''_{\varepsilon=1} \left( \frac{\varepsilon-1}{\varepsilon} \right)^2 \frac{\varepsilon-1}{\varepsilon} \bar{\theta} \frac{1}{\varepsilon} L^{-\frac{1}{\varepsilon}} - (1 + u) A \frac{1}{\varepsilon} F''_{\varepsilon=1} \frac{1}{\varepsilon} \bar{\theta}^{1+\varepsilon} \frac{1}{\varepsilon} L^{-\frac{1}{\varepsilon}} \right] \left[ \begin{array}{c} \frac{dw^+}{d\pi^{op}} \\ \frac{d\pi^{op}}{dL} \\ \frac{d\pi^{op}}{dL} \end{array} \right] = \left[ 0 \right]$$  \hspace{1cm} (21)

According to Cramer’s rule, we find the impact of profit sharing costs on the real basic wage and the number of employees:
\[ \frac{d w^*}{d \frac{\pi^{op}}{L}} = \xi \frac{\bar{e}^{1-1/\bar{\varepsilon}}}{(1 + u) A \frac{1}{\varepsilon} F' \varepsilon^{-1} \bar{e}''^{-2+\varepsilon} \left( \varepsilon^{-1} \bar{e}^{-1} + \bar{e}^{-1} \varepsilon \right) L^{-1/\varepsilon}} > 0 \] (22)

\[ \frac{dL}{d \frac{\pi^{op}}{L}} = -\xi \left[ F'' \varepsilon^{-1} \left( \bar{e}^{-1} \varepsilon \right)^2 L^{-1/\varepsilon} - f' \varepsilon^{-1} \bar{e}''^{-1+\varepsilon} \right] \left( 1 + u \right) A \frac{1}{\varepsilon} F'' \varepsilon^{-1} \bar{e}''^{-1+\varepsilon} \left( \varepsilon^{-1} \bar{e}^{-1} + \bar{e}^{-1} \varepsilon \right)^2 \left( L^{-1/\varepsilon} \right)^2 < 0 \] (23)

Equations (22) and (23) can be expressed in the following elasticities:

\[ \eta_{w} \frac{\pi^{op}}{L} = \varepsilon \frac{1}{\eta_{e} w^*} - \xi \frac{\pi^{op}}{L} > 0 \] (24)

\[ \eta_{L} \frac{\pi^{op}}{L} = \left[ -\varepsilon \frac{1}{\eta_{e} w^*} - \frac{w^*}{\eta_{Q} L} \varepsilon + \frac{\xi \pi^{op}}{L} \right] \left( 1 + u \right) A \frac{1}{\varepsilon} F'' \varepsilon^{-1} \bar{e}''^{-1+\varepsilon} \left( \varepsilon^{-1} \bar{e}^{-1} + \bar{e}^{-1} \varepsilon \right) \left( L^{-1/\varepsilon} \right)^2 < 0 \] (25)

Consistent with the findings of Schmidt-Sorensen, profit sharing depicts a procyclical behaviour with respect to real wage and countercyclical behaviour with respect to labour. The presence of additional labour arrangements increase total labour cost, which in turn would require the firm has to reduce the number of employees. Based on Schmidt-Sorensen’s assumption of fixed employment costs, the firm has to increase the real wage to counteract the resulting reduction of output. The optimal behaviour of the profit-maximizing firm will follow the Law of Increasing Costs: the firm will experience negative profits, ceteris paribus.

Alternatively, the indexation of variable compensation does not display a downward rigidity, as in the case of fixed employment costs, but it creates favourable market conditions where employee displacements and voluntary terminations are significantly reduced. The disaggregation of employee effort and compensation according to the dynamic nature of tasks enable labour to be demonstrated based on firm performance. Whereas the presence of additional fixed labour cost may lead to a non-Pareto-optimal equilibrium and unemployment, variable labour costs may lead to an optimal solution if the total-labour-to-total-cost ratio remains constant: \( \bar{u} \). In periods of economic boom, the firm reaches a Boiteux equilibrium and the total-labour-to-total-cost ratio is a maximum. In periods of economic recession, the total-
labour-to-total-cost ratio reaches a minimum. In the case where the firm exhibits zero or negative profits, the employee receives only the basic wage.

From the equations (18) (19) (24) and (25), profit sharing affects total labour income. As mentioned previously, state intervention can lead to a social optimum by imposing welfare arrangements. The effect on output is found by total differentiation of the production function with respect to variable labour costs, $\xi \frac{\pi_{op}}{L}$, and using equations (24) and (25):

$$\eta_{Q_{op}} = \frac{1}{Q} \frac{1}{\eta_{Q'}} \frac{1}{\frac{\xi}{\eta_{L}} \frac{\xi}{1+\varepsilon} + \frac{\xi}{1+\varepsilon}} > 0$$  \hspace{1cm} (26)

Intuitively, profit sharing move in tandem with output at a different magnitude leading to higher firm and labour productivity, consistent with findings in the empirical literature (Bental & Demougin, 2006; Blasi et al., 2008; Bryson & Freeman, 2008; Carstensen et al., 1995; Conyon & Freeman, 2004; Doucouliagos, 1995; FitzRoy & Kraft, 1987; Kraft & Ugarkovic, 2005; Kruse, 1992; OECD, 1995; Weitzman & Kruse, 1990). In contrast with Schmidt-Sorensen’s fixed employment costs, profit sharing positively affects output. If total-labour-to-cost ratio remains constant, variable labour costs does not affect output neither labour. The aforementioned conclusions imply that the work effort outweighs labour. The number of employees becomes irrelevant.

Finally, the effect on profit is found by total differentiation of (15) with respect to variable labour costs, $\frac{\pi_{op}}{L}$, and using (24) and (25):

$$\eta_{\pi_{op}} = \left[ (1 + u)A \left( \frac{1}{\varepsilon F'} \frac{1}{\varepsilon - 1} \frac{\varepsilon}{\xi} \frac{1}{\xi - \varepsilon'} L - \frac{\varepsilon}{\xi - \varepsilon} - \frac{\xi}{\xi - \varepsilon} \right) \frac{L}{n} > 0 \right]$$  \hspace{1cm} (27)

Similarly, variable labour costs and profit follow the same countercyclical pattern. By keeping variable costs constant in percentage terms, profit sharing component move pro-cyclically with profit. The effect on profit though is greater than the effect on output due to one-to-one relationship. However, the above relationship holds as long as the elasticity marginal product of labour efficiency, $\eta_{Q'L}$, is less or greater than minus one. The elasticity marginal product of labour efficiency lies between minus one and zero. The presence of a CES production function then implies that total labour costs increase (decrease) if variable labour costs, $\xi \frac{\pi_{op}}{L}$, are reduced (increased).

7. **Individual Employee Effort**
The model assumes that all households are identical: they share similar utility preferences and labour effort and income within a constant population growth. The equal distribution of profits may lead to the “free rider” problem (Koskela & Konig, 2011). Employee participation in the form of team-based work structures figures prominently as one of the key driving force for improving firm performance. Nevertheless, teamwork relies on individual effort, raising employee concerns about unobserved heterogeneity. Some of the employees may shirk their responsibilities, if profits are equally distributed because it gives fewer incentives to employees to exert effort. In this case, the effort function is associated with disutility, in the case of present study, it affects both efforts, and can be described by the following convex functions

\[ j(\bar{e}) = \bar{e} (w^*)^{\frac{1}{\varepsilon}}, \quad j'(\bar{e}) = \varepsilon \bar{e} (w^*)^{\frac{1}{\varepsilon} - 1} \]

\[ > 0, \left\{ \begin{array}{ll}
j''(\bar{e}) = \varepsilon \left( \frac{1}{\varepsilon} - 1 \right) \bar{e} (w^*)^{\frac{1}{\varepsilon} - 2} > 0, & \varepsilon < 1 \\
j''(\bar{e}) = \varepsilon \left( \frac{1}{\varepsilon} - 1 \right) \bar{e} (w^*)^{\frac{1}{\varepsilon} - 2} < 0, & \varepsilon > 1 \\
\end{array} \right. \] (28)

\[ j(\hat{e}) = \hat{e} \left( \frac{\pi_{op}}{L} \right)^{\frac{1}{\varepsilon}}, \quad j'(\hat{e}) = \varepsilon \hat{e} \left( \frac{\pi_{op}}{L} \right)^{\frac{1}{\varepsilon} - 1} \]

\[ > 0, \left\{ \begin{array}{ll}
j''(\hat{e}) = \varepsilon \left( \frac{1}{\varepsilon} - 1 \right) \hat{e} \left( \frac{\pi_{op}}{L} \right)^{\frac{1}{\varepsilon} - 2} > 0, & \varepsilon < 1 \\
j''(\hat{e}) = \varepsilon \left( \frac{1}{\varepsilon} - 1 \right) \hat{e} \left( \frac{\pi_{op}}{L} \right)^{\frac{1}{\varepsilon} - 2} < 0, & \varepsilon > 1 \\
\end{array} \right. \] (29)

According to the equity theory (Adams, 1963; Lawler, 1968), employee performance is determined by how employees perceive the fairness of employment relationship based on how they perform, how their performance is rewarded, and how their return-performance ratio is comparable to insiders and outsiders. The optimal effort is determined by setting marginal cost equal to marginal benefit defined by the individual performance, rendering predetermined remuneration irrelevant. This is true independent of the shape of utility over money and the shape of the cost function, conditional on the assumption of separability (Abeler, Falk, Goette, & Huffman, 2011).

6. The Optimal Contract
Measuring individual performance on an on-going basis, offers high effort contracts, a non-Pareto optimal solution. The only way that the firm can induce labour force to exert effort is to offer a proper incentive contract: adjust labour income along firm’s business cycle. The following game tree describes the outcome of an optimal contract.

**Figure 2. Labour incentives**

Source: Author’s elaboration.

There are two possible level of profits for the firm, high $\pi_h$ and low $\pi_l$. Employee effort is affected by the current state of firm profits and he or she can choose to exert high or low effort, affecting the corresponding effort probabilities, $p_h, p_l$, respectively, where $0 < p_l < p_h < 1$. Employee utility is $u(w)$ and the cost of high effort $\xi \frac{\pi^o_p}{L}$ and the cost of low effort $\xi \frac{\pi^o_l}{L}$. If the employee chooses to exert high effort, the corresponding payoff expression is

$$w^*p + (1 - p) \left( p_h \left( u \left( \frac{\pi^o_p}{L} \right)_h - \xi \frac{\pi^o_p}{L} \right) + (1 - p_h) \left( u \left( \frac{\pi^o_l}{L} \right)_l - \xi \frac{\pi^o_l}{L} \right) \right) =$$

$$w^*p + (1 - p) \left( p_h u \left( \frac{\pi^o_p}{L} \right)_h + (1 - p_h) \left( u \left( \frac{\pi^o_p}{L} \right)_h - \xi \frac{\pi^o_p}{L} \right) \right)$$ (30)

Similarly, if the employee chooses to exert low effort, the corresponding payoff expression is
\[ w^*p + (1-p) \left( p_l u \left( \frac{\pi^\text{op}}{L} \right)_h \right) + (1-p_l) \left\{ u \left( \frac{\pi^\text{op}}{L} \right)_h - \xi \frac{\pi^\text{op}}{L} \right\} \] (31)

where \( p \) corresponds to share of basic wage to total labour income. Thus, the employee will choose high effort over low effort only if the first of these expressions is at least as great as the second, which gives the incentive compatibility constraint of eliciting high effort:

\[ (p_h - p_l) \left[ u \left( \frac{\pi^\text{op}}{L} \right)_h \right] \geq \xi \frac{\pi^\text{op}}{L} - \xi \frac{\pi^\text{op}}{L} \] (32)

The employee expects his payoff to be at least equal to the average market wage, \( w^* \):

\[ w^*p + (1-p) \left( p_h u \left( \frac{\pi^\text{op}}{L} \right)_h \right) + (1-p_h) \left\{ u \left( \frac{\pi^\text{op}}{L} \right)_h - \xi \frac{\pi^\text{op}}{L} \right\} \geq \bar{w}^* \] (33)

Equation (33) is the participation constraint under the assumption the employee exerts high effort. Then, the expected profit for the firm will be

\[ p_h \left[ \pi_h - \left( w^* + \xi \frac{\pi^\text{op}}{L} \right) \right] + (1-p_h) \left[ \pi_l - \left( w^* + \xi \frac{\pi^\text{op}}{L} \right) \right] \] (34)

The above equation implies that total labour income should not display a downward rigidity, as the Keynesian theory dictates, but it should fluctuate along with firm profits. The multifaceted nature of the working environment requires the instant adjustment of labour costs. The disaggregation of labour income enhances employment while firms can reduce their break-even point, especially during periods of economic recession. The variable wage is calculated as a percentage of the firm’s operating profit. The relative change in the wage distribution keeps the labour-cost-to-total-cost ratio constant: the basic wage is categorised as a fixed cost, whereas the variable wage is categorised as a variable cost. The total labour income is estimated as follows:

\[ w^*L + \xi y \frac{\pi^\text{op}}{L} (K, L) L = \sum_i w^*_i + \frac{\sum_i \left( w^*_i + \frac{\pi^\text{op}}{L} \right)_i}{BE} \sum_i \left( \frac{EBITDA}{L} \right)_i \] (35)

The labour-cost-to-total-cost ratio \( \xi \) corresponds to the total labour costs to total firm’s costs (explicit/implicit). It equals the sum of basic wages, \( w^*_i \) and \( \frac{\pi^\text{op}}{L} \) variable wage divided to firm’s break-even output, \( BE \) (zero profits).
The coefficient $\xi$ differs for each employee. The operating profit per capita coefficient, $\frac{\eta^{op}}{L}$, represents the ratio of the earnings before interest, taxes, amortisation and depreciation per capita accounting term, $\frac{EBITDA}{L}$. Operating profits measures the operating performance of the firm without taking into account financing (interest payments on debt), political (tax rate), accounting (depreciation) or business (amortisation, and goodwill) factors. Operating income is not used here as a financial metric, but rather as a benchmark for the evaluation of variable labour costs after the deduction of fixed costs (including fixed labour costs). The wage reflects the initial wage, which usually represents the statutory wage, which in turn identifies minimum compensation requirements to perform designated duties and responsibilities of a position. The wage premium acknowledges know-how, skills and competencies acquired through work experience as well as both individual and team performance. The statutory wage and the wage premium commensurate with the levels or hierarchy of the position (entry-level, supervisor, manager). The statutory wage is set by the state, so it is inelastic in the short run. The wage premium variates along with firm profits, so it is perfectly elastic.

7. Concluding Remarks

The profit-share concept in the profit maximisation of a perfect competitive firm provides insights that are helpful in explaining real wage differentials. Like all real efficiency wage models, the equilibrium of the proposed model in this study portrays neutrality: if all exogenous nominal variables change proportionately, then all endogenous nominal variables also change in proportion; output and employment remain unaffected. The efficiency wage elasticity is clearly lower than unity. Profit maximisation thus is not affected by the profit-share factor, and consistent with empirical findings, it can further reduce labour costs.

This research did not receive any financial support from public, private, or not-for-profit sectors.

Notes

1. Variable compensation is highly unlikely to act as a substitute to the base wage due to the “free rider” problem (Holmstrom, 1982; Prendergast, 1999).
2. Inter-market rather than intra-market price competition can also lead to a social optimum (Demsetz, 1968; Williamson, 1976).

4. All formula derivations are in Appendix A.

References


**Appendix**
1. Profit maximization

\[ \max_{w^*, \frac{\pi^0}{L}, L} \pi \left( \bar{\varepsilon}(w^*)L, \bar{\varepsilon} \left( \frac{\pi^0}{L} \right) L \right) = \int_0^\infty \left\{ (1 + u)A_tF^{\varepsilon-1}_t - w^*_tL_t - \frac{\pi^0}{L_t} e^{-rt} \right\} dt \]

Taking the first order conditions with respect to \( w^* \), \( \frac{\pi^0}{L} \), and \( L \)

\[ \frac{d\pi}{dw^*} = (1 + u)A_t \frac{\varepsilon}{\varepsilon-1} F^{\varepsilon-1}_t \frac{\partial}{\partial w^*} \left\{ \left[ \bar{\varepsilon}_t(w^*)L_t \right]^{\frac{\varepsilon-1}{\varepsilon}} \right\} - \frac{\partial}{\partial w^*} w^*_tL_t = 0 \Rightarrow \]

\[ \frac{d\pi}{dw^*} = (1 + u)A_t \frac{\varepsilon}{\varepsilon-1} F^{\varepsilon-1}_t \frac{1}{\varepsilon} L_t \frac{\varepsilon-1}{\varepsilon} \bar{\varepsilon}_t'(w^*) \left( \frac{\varepsilon-1}{\varepsilon} \right) = L_t \frac{1}{\bar{\varepsilon}} \tag{A1} \]

\[ \frac{d\pi}{d\frac{\pi^0}{L}} = (1 + u)A_t \frac{\varepsilon}{\varepsilon-1} F^{\varepsilon-1}_t \frac{1}{\varepsilon} \bar{\varepsilon}_t' \left( \frac{\pi^0}{L} \right) \left( \frac{\varepsilon-1}{\varepsilon} \right) = \xi L_t \Rightarrow \]

\[ (1 + u)A_t \frac{\varepsilon}{\varepsilon-1} F^{\varepsilon-1}_t \frac{1}{\varepsilon} \bar{\varepsilon}_t' \left( \frac{\pi^0}{L} \right) \left( \frac{\varepsilon-1}{\varepsilon} \right) = \xi L_t^\frac{1}{\varepsilon} \tag{A2} \]

\[ (1 + u)A_t \frac{\varepsilon}{\varepsilon-1} F^{\varepsilon-1}_t \frac{1}{\varepsilon} \left\{ \bar{\varepsilon}_t(w^*)L_t \frac{\varepsilon-1}{\varepsilon} + \left[ \bar{\varepsilon}_t \left( \frac{\pi^0}{L} \right) L_t \right] \frac{\varepsilon-1}{\varepsilon} \right\} - \frac{\partial}{\partial L} w^*_tL_t - \xi \frac{\partial}{\partial L} \frac{\pi^0}{L_t} L_t = 0 \Rightarrow \]

\[ (1 + u)A_t \frac{\varepsilon}{\varepsilon-1} F^{\varepsilon-1}_t \frac{1}{\varepsilon} \left[ \bar{\varepsilon}_t(w^*) \frac{\varepsilon-1}{\varepsilon} + \bar{\varepsilon}_t \left( \frac{\pi^0}{L} \right) \frac{\varepsilon-1}{\varepsilon} L_t \frac{\varepsilon-1}{\varepsilon} \right] - w^*_t - \xi \frac{\pi^0}{L_t} = 0 \Rightarrow \]

\[ (1 + u)A_t \frac{\varepsilon}{\varepsilon-1} F^{\varepsilon-1}_t \frac{1}{\varepsilon} = \frac{w^*_t - \bar{\varepsilon}_t \left( \frac{\pi^0}{L} \right) \frac{\varepsilon-1}{\varepsilon} L_t^\frac{1}{\varepsilon}}{\bar{\varepsilon}(w^*) + \bar{\varepsilon}_t \left( \frac{\pi^0}{L} \right)^{\frac{\varepsilon-1}{\varepsilon}}} \tag{A3} \]

2. Derivations of the Primary Elasticities
Inserting equation (16) into equation (14) give

\[
\tilde{e}'(w^*) \frac{1}{\varepsilon} \frac{w^* + \xi^{\text{nop}}}{[\tilde{e}(w^*)]^{\varepsilon\varepsilon} + \tilde{e}(\xi^{\text{nop}})} \varepsilon = 1 \Rightarrow \tilde{e}'(w^*) \frac{1}{\varepsilon} \frac{w^* + \xi^{\text{nop}}}{[\tilde{e}(w^*)]^{\varepsilon\varepsilon} + \tilde{e}(\xi^{\text{nop}})} \varepsilon = 1 \Rightarrow \\
\eta \tilde{e} \left( \frac{1 + \xi^{\text{nop}}}{w^*} \right) = 1
\]

(A4)

Similarly, insertion of equation (16) into equation (14) give

\[
\tilde{e}' \left( \frac{\pi^{\text{nop}}}{L} \right) \frac{1}{\varepsilon} \frac{w^* + \xi^{\text{nop}}}{[\tilde{e}(w^*)]^{\varepsilon\varepsilon} + \tilde{e}(\xi^{\text{nop}})} \varepsilon = \xi \Rightarrow \\
\tilde{e}' \left( \frac{\pi^{\text{nop}}}{L} \right) \frac{1}{\varepsilon} \frac{w^* + \xi^{\text{nop}}}{[\tilde{e}(w^*)]^{\varepsilon\varepsilon} + \tilde{e}(\xi^{\text{nop}})} \varepsilon = \xi \Rightarrow \\
\eta \tilde{e} \left( \frac{1 + \frac{w^*}{\xi^{\text{nop}}}}{L} \right) = \xi < 1
\]

(A5)

3. Hessian Matrix

The second order derivatives with respect $w^*$ and $L$ takes the Hessian matrix form $H = \begin{bmatrix} \pi \pi & \pi \pi \\ \pi \pi & \pi \pi \end{bmatrix}$ and examine the impact of $\frac{\pi^{\text{nop}}}{L}$. Let $F = \begin{bmatrix} \pi \pi \\ \pi \pi \end{bmatrix}$ and $\tilde{e}'(w^*)$ and $\tilde{e} = \tilde{e} \left( \frac{\pi^{\text{nop}}}{L} \right)$. Equation (14) takes the form $(1 + u)A F' \frac{1}{\varepsilon - 1} \tilde{e}' \frac{1}{\varepsilon} L^{-1} \frac{1}{\varepsilon} = 1$. The partial derivatives of the equation (14) in respect to $w^*, L$ and $\frac{\pi^{\text{nop}}}{L}$ give

\[
(1 + u)A F' \frac{1}{\varepsilon - 1} \tilde{e}' \frac{1}{\varepsilon} L^{-1} \frac{1}{\varepsilon} = 1 \Rightarrow \\
d \left( \frac{d\pi}{d\pi} \right) = (1 + u) A \tilde{e}' \frac{1}{\varepsilon} L^{-1} \frac{1}{\varepsilon} F' \frac{1}{\varepsilon - 1} L^{-1} \frac{1}{\varepsilon} \frac{\partial \varepsilon}{\partial w^*} \frac{1}{\varepsilon} d\pi + \\
\frac{d\pi}{d\pi} = (1 + u) A \tilde{e}' \frac{1}{\varepsilon} L^{-1} \frac{1}{\varepsilon} F' \frac{1}{\varepsilon - 1} L^{-1} \frac{1}{\varepsilon} \frac{\partial \varepsilon}{\partial w^*} \frac{1}{\varepsilon} d\pi + \\
\frac{d\pi}{d\pi} = (1 + u) A \tilde{e}' \frac{1}{\varepsilon} L^{-1} \frac{1}{\varepsilon} F' \frac{1}{\varepsilon - 1} L^{-1} \frac{1}{\varepsilon} \frac{\partial \varepsilon}{\partial w^*} \frac{1}{\varepsilon} d\pi + \\
\frac{d\pi}{d\pi} = (1 + u) A \tilde{e}' \frac{1}{\varepsilon} L^{-1} \frac{1}{\varepsilon} F' \frac{1}{\varepsilon - 1} L^{-1} \frac{1}{\varepsilon} \frac{\partial \varepsilon}{\partial w^*} \frac{1}{\varepsilon} d\pi +
\]
\[(1 + u)AF'\frac{1}{\varepsilon - 1}L\frac{1}{\varepsilon} \frac{\partial}{\partial w^*} \bar{\varepsilon}'\frac{1}{\varepsilon}dw^* + \]

\[(1 + u)A \bar{e}'\frac{1}{\varepsilon} L^{-\frac{1}{\varepsilon}} \frac{1}{\varepsilon - 1}F''\frac{1}{\varepsilon - 1} - 1 \frac{\partial}{\partial L} f dL = \frac{\partial}{\partial L} \frac{\pi_{op}}{L} 1 d \frac{\pi_{op}}{L} \Rightarrow \]

\[d \left(\frac{d\pi}{dw^*}\right) = (1 + u)A \bar{e}'\frac{1}{\varepsilon} L^{-\frac{1}{\varepsilon}} \frac{1}{\varepsilon - 1}F''\frac{2-\varepsilon}{\varepsilon - 1} \frac{1}{\varepsilon} L^\frac{1}{\varepsilon - 1} - \frac{\varepsilon - 1}{\varepsilon} - 1 \frac{\partial}{\partial w^*} \bar{\varepsilon}'\frac{1}{\varepsilon} \frac{1}{\varepsilon - 1}dw^* + \]

\[(1 + u)A F'\frac{1}{\varepsilon - 1}L^{-\frac{1}{\varepsilon}} \left(-\frac{1}{\varepsilon}\right) \bar{e}'\frac{1}{\varepsilon} \frac{1}{\varepsilon - 1}dw^* + (1 + u)A \bar{e}'\frac{1}{\varepsilon} L^{-\frac{1}{\varepsilon}} \frac{1}{\varepsilon - 1} \Rightarrow \]

\[d \left(\frac{d\pi}{dw^*}\right) = \left[(1 + u)A \frac{1}{\varepsilon} F''\frac{1}{\varepsilon - 1} \left(\bar{e}'\frac{1}{\varepsilon} \frac{1}{\varepsilon - 1} + \bar{e}'\frac{1}{\varepsilon} \frac{1}{\varepsilon - 1} \right) \right] \frac{\pi_{op}}{L} = 0 \]

\[(1 + u)A \frac{1}{\varepsilon} F''\frac{1}{\varepsilon - 1} \bar{e}'\frac{1}{\varepsilon} \frac{1+\varepsilon}{\varepsilon} L^{-\frac{1}{\varepsilon}} \frac{1}{\varepsilon} \frac{\pi_{op}}{L} = 0 \quad (1) \]

Accordingly, the partial derivatives of the equation (16) in respect to \(w^*, L\) and \(\frac{\pi_{op}}{L}\) give

\[(1 + u)AF'\frac{1}{\varepsilon - 1} \left(\bar{e}'\frac{1}{\varepsilon} + \bar{e}'\frac{1}{\varepsilon}\right) L^{-\frac{1}{\varepsilon}} = w^* + \xi \frac{\pi_{op}}{L} \Rightarrow \]

\[d \left(\frac{d\pi}{dL}\right) = (1 + u)A \left(\bar{e}'\frac{1}{\varepsilon} + \bar{e}'\frac{1}{\varepsilon}\right) L^{-\frac{1}{\varepsilon}} \frac{1}{\varepsilon - 1} F''\frac{1}{\varepsilon - 1} - 1 \frac{\partial}{\partial w^*} \bar{\varepsilon}'\frac{1}{\varepsilon} \frac{1}{\varepsilon - 1}dw^* + \]

\[(1 + u)A \left(\bar{e}'\frac{1}{\varepsilon} + \bar{e}'\frac{1}{\varepsilon}\right) L^{-\frac{1}{\varepsilon}} \frac{1}{\varepsilon - 1} F''\frac{1}{\varepsilon - 1} - 1 \frac{\partial}{\partial L} \left[(\varepsilon L)^{\frac{1}{\varepsilon}} + (\varepsilon L)^{\frac{1}{\varepsilon}}\right] dL = \]
\[
\frac{\partial}{\partial \pi_{op}} \left[ \pi_{op} \frac{w^*}{L} \right] + \xi \frac{\partial}{\partial \pi_{op}} \left[ \pi_{op} \frac{d L}{L} \right] = 0
\]

\[
d \left( \frac{d \pi}{d L} \right) = (1 + u) A \left( \pi \frac{\varepsilon - 1}{\varepsilon} + \pi \frac{\varepsilon - 1}{\varepsilon} \right) L_{\varepsilon - 1}^{1} e_{\varepsilon - 1}^{2 - \varepsilon} L_{\varepsilon - 1}^{\varepsilon - 1} d w^* + \\
(1 + u) A \left( \pi \frac{\varepsilon - 1}{\varepsilon} + \pi \frac{\varepsilon - 1}{\varepsilon} \right) L_{\varepsilon - 1}^{1} e_{\varepsilon - 1}^{2 - \varepsilon} L_{\varepsilon - 1}^{\varepsilon - 1} d L = 0 + \xi d \frac{\pi_{op}}{L} \\

d \left( \frac{d \pi}{d L} \right) = (1 + u) A \left( \pi \frac{\varepsilon - 1}{\varepsilon} + \pi \frac{\varepsilon - 1}{\varepsilon} \right) L_{\varepsilon - 1}^{1} e_{\varepsilon - 1}^{2 - \varepsilon} L_{\varepsilon - 1}^{\varepsilon - 1} d w^* + \\
(1 + u) A \left( \pi \frac{\varepsilon - 1}{\varepsilon} + \pi \frac{\varepsilon - 1}{\varepsilon} \right) L_{\varepsilon - 1}^{1} e_{\varepsilon - 1}^{2 - \varepsilon} L_{\varepsilon - 1}^{\varepsilon - 1} d L = \xi d \frac{\pi_{op}}{L}
\]

(2)

The matrix form of equations (1) and (2)

\[
\begin{bmatrix}
(1 + u) A \left( \frac{\varepsilon - 1}{\varepsilon} \right) L_{\varepsilon - 1}^{1} e_{\varepsilon - 1}^{2 - \varepsilon} L_{\varepsilon - 1}^{\varepsilon - 1} \\
(1 + u) A \left( \frac{\varepsilon - 1}{\varepsilon} \right) L_{\varepsilon - 1}^{1} e_{\varepsilon - 1}^{2 - \varepsilon} L_{\varepsilon - 1}^{\varepsilon - 1} \\
(1 + u) A \left( \frac{\varepsilon - 1}{\varepsilon} \right) L_{\varepsilon - 1}^{1} e_{\varepsilon - 1}^{2 - \varepsilon} L_{\varepsilon - 1}^{\varepsilon - 1}
\end{bmatrix}
= \begin{bmatrix}
\frac{d w^*}{d \pi_{op}} \\
\frac{d L}{d \pi_{op}} \\
\frac{d L}{d \pi_{op}}
\end{bmatrix} = \begin{bmatrix}
\xi
\end{bmatrix} (A6)
\]
Applying Cramer’s rule, we need to find \( \frac{dw^*}{d\omega^p_L} = \frac{|A_1|}{|A|} \) and \( \frac{dl}{d\omega^p_L} = \frac{|A_2|}{|A|} \). First, we calculate the determinants \( |A_1|, |A_2| \) and \( |A| \):

\[
|A_1| = \left[ \begin{array}{c} 0 - \xi (1 + u) A \frac{1}{\varepsilon} F''_{\varepsilon-1} (\varepsilon^{-1}) \left( \frac{\varepsilon - 1}{\varepsilon} + \frac{\varepsilon - 1}{\varepsilon} \right) \left( L^{-1} \right)^2 \end{array} \right] 
\]

\[
|A_1| = 0 - \xi (1 + u) A \frac{1}{\varepsilon} F''_{\varepsilon-1} (\varepsilon^{-1}) \left( \frac{\varepsilon - 1}{\varepsilon} + \frac{\varepsilon - 1}{\varepsilon} \right) \left( L^{-1} \right)^2 \Rightarrow 
\]

\[
|A_1| = -\xi (1 + u) A \frac{1}{\varepsilon} F''_{\varepsilon-1} (\varepsilon^{-1}) \left( \frac{\varepsilon - 1}{\varepsilon} + \frac{\varepsilon - 1}{\varepsilon} \right) \left( L^{-1} \right)^2 < (A7) 
\]

\[
|A_2| = \left[ \begin{array}{c} (1 + u) A \frac{1}{\varepsilon} F''_{\varepsilon-1} (\varepsilon^{-1}) \left( \frac{\varepsilon - 1}{\varepsilon} \right) L^{-1} - (1 + u) A \frac{1}{\varepsilon} F'_{\varepsilon-1} (\varepsilon^{-1}) L^{-1} 0 \end{array} \right] 
\]

\[
(1 + u) A \frac{1}{\varepsilon} F'_{\varepsilon-1} (\varepsilon^{-1}) L^{-1} \Rightarrow 
\]

\[
|A_2| = \xi (1 + u) A \frac{1}{\varepsilon} F''_{\varepsilon-1} (\varepsilon^{-1}) \left( \frac{\varepsilon - 1}{\varepsilon} \right) L^{-1} > 0 (A8) 
\]

\[
|A| = \left[ \begin{array}{c} (1 + u) A \frac{1}{\varepsilon} F''_{\varepsilon-1} (\varepsilon^{-1}) \left( \frac{\varepsilon - 1}{\varepsilon} \right) L^{-1} - (1 + u) A \frac{1}{\varepsilon} F'_{\varepsilon-1} (\varepsilon^{-1}) L^{-1} \end{array} \right] 
\]

\[
(1 + u) A \frac{1}{\varepsilon} F''_{\varepsilon-1} (\varepsilon^{-1}) \left( \frac{\varepsilon - 1}{\varepsilon} \right) L^{-1} \Rightarrow 
\]

\[
(1 + u) A \frac{1}{\varepsilon} F'_{\varepsilon-1} (\varepsilon^{-1}) L^{-1}^2 \]

\[
|A| = \left[ (1 + u) A \frac{1}{\varepsilon} F'' \frac{2-\varepsilon}{\varepsilon-1} \left( e^{-1 \varepsilon} \right)^2 \frac{L^{-1} \varepsilon}{L^{-1} \varepsilon} - (1 + u) A \frac{1}{\varepsilon} F' \frac{1}{\varepsilon-1} e^{-1+\varepsilon \frac{1}{L} \varepsilon} \right] \\
(1 + u) A \frac{1}{\varepsilon} \left( e^{\frac{\varepsilon-1}{\varepsilon}} + e^{\frac{\varepsilon-1}{\varepsilon}} \right) F'' \frac{2-\varepsilon}{\varepsilon-1} \left( L^{-1} \varepsilon \right)^2 - (1 + u) A \frac{1}{\varepsilon} F'' \frac{2-\varepsilon}{\varepsilon-1} e^{\frac{1}{L} \varepsilon} \\
\left( e^{\frac{\varepsilon-1}{\varepsilon}} + e^{\frac{\varepsilon-1}{\varepsilon}} \right) \left( L^{-1} \varepsilon \right)^2 (1 + u) A \frac{1}{\varepsilon} \left( e^{\frac{\varepsilon-1}{\varepsilon}} + e^{\frac{\varepsilon-1}{\varepsilon}} \right) F'' \frac{2-\varepsilon}{\varepsilon-1} \left( L^{-1} \varepsilon \right)^2 \\
(1 + u) A \frac{1}{\varepsilon} F'' \frac{2-\varepsilon}{\varepsilon-1} e^{\frac{1}{L} \varepsilon} \left( \varepsilon^{\frac{1}{\varepsilon}} + e^{-\frac{1}{\varepsilon}} \right) \left( L^{-1} \varepsilon \right)^2 \\
(1 + u) A \frac{1}{\varepsilon} \left( e^{\frac{\varepsilon-1}{\varepsilon}} + e^{\frac{\varepsilon-1}{\varepsilon}} \right) F'' \frac{2-\varepsilon}{\varepsilon-1} e^{\frac{1}{L} \varepsilon} \frac{L^{-1} \varepsilon}{L^{-1} \varepsilon} \\
|A| = (1 + u)^2 A^2 \left( \frac{1}{\varepsilon} \right)^2 \left( F'' \frac{2-\varepsilon}{\varepsilon-1} \right)^2 \left( \varepsilon^{\frac{1}{\varepsilon}} + e^{\frac{\varepsilon-1}{\varepsilon}} \right)^2 \frac{L^{-1} \varepsilon}{L^{-1} \varepsilon} \left( L^{-1} \varepsilon \right)^3 \\
(1 + u)^2 A^2 \left( \frac{1}{\varepsilon} \right)^2 \left( F'' \frac{2-\varepsilon}{\varepsilon-1} \right) F' \frac{1}{\varepsilon-1} e^{\frac{1+\varepsilon}{\varepsilon}} e^{-\frac{1}{\varepsilon}} \frac{L^{-1} \varepsilon}{L^{-1} \varepsilon} \\
(1 + u)^2 A^2 \left( \frac{1}{\varepsilon} \right)^2 \left( F'' \frac{2-\varepsilon}{\varepsilon-1} \right) \left( \varepsilon^{\frac{1}{\varepsilon}} + e^{\frac{\varepsilon-1}{\varepsilon}} \right)^2 \left( L^{-1} \varepsilon \right)^3 \\
|A| = -(1 + u)^2 A^2 \left( \frac{1}{\varepsilon} \right)^2 \left( F'' \frac{2-\varepsilon}{\varepsilon-1} \right) F' \frac{1}{\varepsilon-1} e^{\frac{1+\varepsilon}{\varepsilon}} e^{-\frac{1}{\varepsilon}} \left( e^{\frac{\varepsilon-1}{\varepsilon}} + e^{\frac{\varepsilon-1}{\varepsilon}} \right)^2 \left( L^{-1} \varepsilon \right)^3 > 0 \quad (A9) \\
\text{Then we have} \\
\frac{dw^*}{d \pi_0 L} = \frac{|A_1|}{|A|} 
\]
\[ -\xi (1 + u) \frac{1}{\varepsilon} F'' \left( \frac{2-\varepsilon}{\varepsilon-1} \frac{1}{\varepsilon} \left( \frac{\varepsilon-1}{\varepsilon} + \hat{\varepsilon} \right) \left( \frac{1}{\varepsilon} - \frac{1}{\varepsilon} \right)^2 \right) = -(1 + u)^2 \frac{A^2}{\varepsilon^2} \left( \frac{1}{\varepsilon} \right)^2 \left( F'' \varepsilon - 1 \frac{1}{\varepsilon} \varepsilon - F' \varepsilon - 1 \frac{1}{\varepsilon} \varepsilon + 1 + \varepsilon \right) L \left( \frac{1}{\varepsilon} \right) \]

\[ \frac{dw^*}{d \frac{\pi_{op}}{L}} = \xi \frac{\hat{\varepsilon}^{-1} \frac{1}{\varepsilon}}{(1 + u) A \frac{1}{\varepsilon} F' \frac{1}{\varepsilon} - 1 \left( \frac{1}{\varepsilon} \frac{1}{\varepsilon} \left( \frac{1}{\varepsilon} + \varepsilon - 1 \right) \right) L \frac{1}{\varepsilon} > 0 \quad (A10) \]

\[ \frac{dL}{d \frac{\pi_{op}}{L}} = \frac{|A_2|}{|A|} = \xi (1 + u) \frac{A^2}{\varepsilon^2} \left( \frac{1}{\varepsilon} \right)^2 \left( F'' \varepsilon - 1 \frac{1}{\varepsilon} \varepsilon - F' \varepsilon - 1 \frac{1}{\varepsilon} \varepsilon + 1 + \varepsilon \right) L \left( \frac{1}{\varepsilon} \right) \]

\[ \frac{dL}{d \frac{\pi_{op}}{L}} = - \xi \left( \frac{2-\varepsilon}{\varepsilon} \frac{1}{\varepsilon} \varepsilon - F' \varepsilon - 1 \frac{1}{\varepsilon} \varepsilon + 1 + \varepsilon \right) \left( \frac{1}{\varepsilon} \frac{1}{\varepsilon} \left( \varepsilon - 1 \right)^2 \left( \frac{1}{\varepsilon} - \frac{1}{\varepsilon} \right)^2 \right) \frac{L}{\varepsilon} < 0 \quad (A11) \]

### 4. Derivation of the secondary elasticities

\[ \frac{dw^*}{d \frac{\pi_{op}}{L}} = \xi \frac{\hat{\varepsilon}^{-1} \frac{1}{\varepsilon}}{(1 + u) A \frac{1}{\varepsilon} F' \frac{1}{\varepsilon} - 1 \frac{1}{\varepsilon} \varepsilon - 1 + \varepsilon \left( \frac{1}{\varepsilon} + \varepsilon - 1 \right) \right) L \frac{1}{\varepsilon} } \]

Insert equation (14) \((1 + u) A F' \frac{1}{\varepsilon} - 1 L \frac{1}{\varepsilon} \varepsilon \frac{1}{\varepsilon} = 1 \Rightarrow (1 + u) A F' \frac{1}{\varepsilon} - 1 L \frac{1}{\varepsilon} = 1 \frac{1}{\varepsilon} - \frac{1}{\varepsilon} \)

\[ \frac{dw^*}{d \frac{\pi_{op}}{L}} = \xi \frac{\frac{1}{\varepsilon} \frac{1}{\varepsilon} \varepsilon - 1 + \varepsilon \left( \frac{1}{\varepsilon} + \varepsilon - 1 \right) \right) \xi \frac{\frac{1}{\varepsilon} \frac{1}{\varepsilon} \frac{1}{\varepsilon}}{\varepsilon - 1 + \varepsilon \left( \frac{1}{\varepsilon} + \varepsilon - 1 \right) \right) \Rightarrow \]

\[ \frac{dw^*}{d \frac{\pi_{op}}{L}} = \xi \frac{\frac{1}{\varepsilon} \frac{1}{\varepsilon} \varepsilon - 1 + \varepsilon \left( \frac{1}{\varepsilon} + \varepsilon - 1 \right) \right) \left( \frac{1}{\varepsilon} \frac{1}{\varepsilon} \frac{1}{\varepsilon} \right) \]

\[ \frac{dw^*}{d \frac{\pi_{op}}{L}} = \xi \frac{\frac{1}{\varepsilon} \frac{1}{\varepsilon} \varepsilon - 1 + \varepsilon \left( \frac{1}{\varepsilon} + \varepsilon - 1 \right) \right) \frac{1}{\varepsilon} \frac{1}{\varepsilon} \frac{1}{\varepsilon} \]

\[ \frac{dw^*}{d \frac{\pi_{op}}{L}} = \xi \frac{\frac{1}{\varepsilon} \frac{1}{\varepsilon} \varepsilon - 1 + \varepsilon \left( \frac{1}{\varepsilon} + \varepsilon - 1 \right) \right) \frac{1}{\varepsilon} \frac{1}{\varepsilon} \frac{1}{\varepsilon} \]

\[ \frac{dw^*}{d \frac{\pi_{op}}{L}} = \xi \frac{\frac{1}{\varepsilon} \frac{1}{\varepsilon} \varepsilon - 1 + \varepsilon \left( \frac{1}{\varepsilon} + \varepsilon - 1 \right) \right) \frac{1}{\varepsilon} \frac{1}{\varepsilon} \frac{1}{\varepsilon} \]

\[ \frac{dw^*}{d \frac{\pi_{op}}{L}} = \xi \frac{\frac{1}{\varepsilon} \frac{1}{\varepsilon} \varepsilon - 1 + \varepsilon \left( \frac{1}{\varepsilon} + \varepsilon - 1 \right) \right) \frac{1}{\varepsilon} \frac{1}{\varepsilon} \frac{1}{\varepsilon} \]
The combination of equations (16) and (14) gives

\[
\begin{align*}
\frac{a \tilde{e}'(w^*)}{w^* + \xi \pi^{op}_L} \frac{1}{\varepsilon} \left[ \tilde{e}'(w^*) \right]^{\frac{\varepsilon-1}{\varepsilon}} + \left[ \tilde{e}' \left( \frac{\pi^{op}_L}{L} \right) \right]^{\frac{\varepsilon-1}{\varepsilon}} = 1 \\
\Rightarrow \frac{\tilde{e}'(w^*)}{[\tilde{e}(w^*)]^{\frac{\varepsilon-1}{\varepsilon}} + \left[ \tilde{e}' \left( \frac{\pi^{op}_L}{L} \right) \right]^{\frac{\varepsilon-1}{\varepsilon}}} = \frac{1}{w^* + \xi \pi^{op}_L}.
\end{align*}
\]

Then (b) takes the form

\[
\frac{d w^*}{d \pi^{op}_L} = \xi \varepsilon \frac{1}{\tilde{e}' \left( \frac{1+\varepsilon}{\varepsilon} \right)} \frac{1}{w^* + \xi \pi^{op}_L}.
\]

Multiply both parts with \( \frac{\pi^{op}_L}{w^*} \) and rearrange the first and second effort derivatives

\[
\frac{d w^*}{d \pi^{op}_L} \frac{\pi^{op}_L}{w^*} = \xi \varepsilon \frac{1}{\tilde{e}' \left( \frac{1+\varepsilon}{\varepsilon} \right)} \frac{1}{w^* + \xi \pi^{op}_L} \Rightarrow
\]

\[
\eta_{\pi^{op}_L} = \varepsilon \frac{1}{\eta_{\tilde{e}'w^*}} \frac{\xi \pi^{op}_L}{w^* + \xi \pi^{op}_L} > 0 \quad (A12)
\]

\[
\frac{d L}{d \pi^{op}_L} = -\frac{\xi}{(1 + u) A} \left( \frac{1}{\varepsilon} \right)^{2} \frac{\tilde{e}' \left( \frac{1+\varepsilon}{\varepsilon} \right)^{2} \left( \frac{\varepsilon-1}{\varepsilon} \right)^{2}}{L} \left( \frac{\tilde{e}' + \tilde{e}'}{\varepsilon} \right)^{2} \left( L \frac{1}{\varepsilon} \right)^{2}
\]

\[
\frac{d L}{d \pi^{op}_L} = -\frac{\xi}{(1 + u) A} \left( \frac{1}{\varepsilon} \right)^{2} \frac{\tilde{e}' \left( \frac{1+\varepsilon}{\varepsilon} \right)^{2} \left( \frac{\varepsilon-1}{\varepsilon} \right)^{2}}{L} \left( \tilde{e}' + \tilde{e}' \right)^{2} \left( L \frac{1}{\varepsilon} \right)^{2}
\]
\[
(1 + u) A \frac{1}{\varepsilon} F'' \frac{2-\varepsilon}{\varepsilon-1} \frac{1}{\varepsilon-1} \frac{1}{\varepsilon-1} e'' \left( \frac{e^{-1}}{e} + \hat{e}^{-1} \right) \left( L^{-1} \frac{1}{e} \right)^2 \Rightarrow \\
\frac{dL}{d \pi_{op}} = - \frac{\xi}{1 + u} A \frac{1}{\varepsilon} F'' \frac{2-\varepsilon}{\varepsilon-1} \frac{1}{\varepsilon-1} e'' \left( \frac{e^{-1}}{e} + \hat{e}^{-1} \right) \left( L^{-1} \frac{1}{e} \right)^2 \]

\[
(1 + u) AF \frac{1}{\varepsilon-1} L \frac{1}{\varepsilon} e^{-1} \frac{1}{e} = 1 \Rightarrow (1 + u) AF \frac{1}{\varepsilon-1} L \frac{1}{\varepsilon} = \frac{1}{\hat{e}^{-1} \varepsilon} \text{ and} \\
(1 + u) AF \frac{1}{\varepsilon-1} L \frac{1}{\varepsilon} = \frac{1}{\hat{e}^{-1} \varepsilon} \\
\frac{dL}{d \pi_{op}} = - \frac{\xi}{\hat{e}^{-1} \varepsilon} F' \frac{2-\varepsilon}{\varepsilon-1} \left( \frac{e^{-1}}{e} + \hat{e}^{-1} \right) \left( L^{-1} \frac{1}{e} \right)^2 \]

\[
\frac{\xi}{\hat{e}^{-1} \varepsilon} F' \frac{2-\varepsilon}{\varepsilon-1} \left( \frac{e^{-1}}{e} + \hat{e}^{-1} \right) \left( L^{-1} \frac{1}{e} \right)^2 \Rightarrow \\
\frac{dL}{d \pi_{op}} = - \xi \varepsilon L L \frac{1}{\varepsilon} \frac{1}{\hat{e}^{-1} \varepsilon} \frac{1}{\hat{e}^{-1} \varepsilon} \frac{1}{\hat{e}^{-1} \varepsilon} \left( \frac{e^{-1}}{e} + \hat{e}^{-1} \right) \left( L^{-1} \frac{1}{e} \right)^2 \]

\[
\frac{\xi \varepsilon}{\hat{e}^{-1} \varepsilon} F' \frac{2-\varepsilon}{\varepsilon-1} \left( \frac{e^{-1}}{e} + \hat{e}^{-1} \right) \left( L^{-1} \frac{1}{e} \right)^2 + \\
\frac{1}{F' \frac{2-\varepsilon}{\varepsilon-1}} \frac{1}{\hat{e}^{-1} \varepsilon} \frac{1}{\hat{e}^{-1} \varepsilon} \frac{1}{\hat{e}^{-1} \varepsilon} \left( \frac{e^{-1}}{e} + \hat{e}^{-1} \varepsilon \right) \left( L^{-1} \frac{1}{e} \right)^2 \]

\[
\xi \varepsilon \frac{1}{F' \frac{2-\varepsilon}{\varepsilon-1}} \frac{1}{\hat{e}^{-1} \varepsilon} \frac{1}{\hat{e}^{-1} \varepsilon} \frac{1}{\hat{e}^{-1} \varepsilon} \left( \frac{e^{-1}}{e} + \hat{e}^{-1} \varepsilon \right) \left( L^{-1} \frac{1}{e} \right)^2 \]

\[
(c) \]
The combination of equations (16) and (15) gives

\[
\frac{a \, \bar{e}'(w^*)^{\frac{1}{\varepsilon}} \, w^* + \xi \, \frac{\pi_{op}}{L}}{[\bar{e}(w^*)]^{\frac{\varepsilon-1}{\varepsilon}} + [\bar{e}(\frac{\pi_{op}}{L})]^{\frac{\varepsilon-1}{\varepsilon}}} = 1
\]

\[
\Rightarrow \frac{a \, \bar{e}'(w^*)^{\frac{1}{\varepsilon}}}{a[\bar{e}(w^*)]^{\frac{\varepsilon-1}{\varepsilon}} + \beta[\bar{e}(\frac{\pi_{op}}{L})]^{\frac{\varepsilon-1}{\varepsilon}}} = \frac{1}{w^* + \xi \, \frac{\pi_{op}}{L}}
\]

Then (c) takes the form

\[
\frac{dL}{d \, \frac{\pi_{op}}{L}} = -\xi \varepsilon L \, \frac{\bar{e}' - \frac{1}{\varepsilon}}{\bar{e}'' - \frac{1+\varepsilon}{\varepsilon} \, \left( w^* + \xi \, \frac{\pi_{op}}{L} \right)^2} + \frac{1}{L} \, \frac{\varepsilon}{\bar{e}'' - \frac{1+\varepsilon}{\varepsilon}} \left( w^* + \xi \, \frac{\pi_{op}}{L} \right) \frac{1}{\bar{e}' - \frac{1}{\varepsilon}} + \bar{e} \frac{\varepsilon}{\varepsilon - 1} w^* + \xi \, \frac{\pi_{op}}{L}
\]

Take out the common factor:

\[
\frac{dL}{d \, \frac{\pi_{op}}{L}} = -\frac{\bar{e}' - \frac{1}{\varepsilon}}{\bar{e}'' - \frac{1+\varepsilon}{\varepsilon} \, w^* + \xi \, \frac{\pi_{op}}{L}} + \frac{1}{L} \, \frac{\varepsilon}{\bar{e}'' - \frac{1+\varepsilon}{\varepsilon}} \, \frac{1}{\bar{e}' - \frac{1}{\varepsilon}} \, \frac{1}{\bar{e}'' - \frac{1+\varepsilon}{\varepsilon} \, w^* + \xi \, \frac{\pi_{op}}{L}} L \, \xi \varepsilon \, \frac{1}{w^* + \xi \, \frac{\pi_{op}}{L}} \Rightarrow
\]

Send \( L \) on the other side and multiply with \( \frac{w^*}{w^*} \) the first fraction of the bracket:

\[
\frac{dL}{d \, \frac{\pi_{op}}{L}} = -\frac{\bar{e}' - \frac{1}{\varepsilon}}{\bar{e}'' - \frac{1+\varepsilon}{\varepsilon} \, w^* + \xi \, \frac{\pi_{op}}{L}} \frac{1}{w^*} + \frac{1}{L} \, \frac{\varepsilon}{\bar{e}'' - \frac{1+\varepsilon}{\varepsilon}} \, \frac{1}{\bar{e}' - \frac{1}{\varepsilon}} \, \frac{1}{\bar{e}'' - \frac{1+\varepsilon}{\varepsilon} \, w^* + \xi \, \frac{\pi_{op}}{L}} \frac{1}{L} \, \frac{\varepsilon}{\bar{e}'' - \frac{1+\varepsilon}{\varepsilon}} \, \frac{1}{\bar{e}' - \frac{1}{\varepsilon}} \, \frac{1}{\bar{e}'' - \frac{1+\varepsilon}{\varepsilon} \, w^* + \xi \, \frac{\pi_{op}}{L}} L \, \xi \varepsilon \, \frac{1}{w^* + \xi \, \frac{\pi_{op}}{L}} \Rightarrow
\]
\[ \frac{1}{w^* + \xi \frac{\pi_{op}}{L}} \Rightarrow \]

Multiply by \( \frac{\pi_{op}}{L} \) both sides of the equation:

\[ \frac{dL}{d \frac{\pi_{op}}{L}} \frac{\pi_{op}}{L} = \left[ -\frac{\tilde{e}^{\prime \prime} - \frac{1}{\tilde{e}} w^*}{\tilde{e}^{\prime \prime} - \frac{1+\tilde{e}}{\tilde{e}} w^*} + \frac{1}{\tilde{e}^{\prime \prime} - \frac{2-\tilde{e}}{\tilde{e}} \frac{\pi_{op}}{L}} + \frac{1}{1} \right] \left[ 1 \right] \]

\[ \xi \varepsilon \frac{\pi_{op}}{L} \Rightarrow \]

\[ \frac{dL}{d \frac{\pi_{op}}{L}} \frac{\pi_{op}}{L} = \]

\[ \left[ -\frac{\tilde{e}^{\prime \prime} - \frac{1}{\tilde{e}} w^*}{\tilde{e}^{\prime \prime} - \frac{1+\tilde{e}}{\tilde{e}} w^*} + \frac{1}{\tilde{e}^{\prime \prime} - \frac{2-\tilde{e}}{\tilde{e}} \frac{\pi_{op}}{L}} + \frac{1}{1} \right] \left[ \xi \varepsilon \frac{\pi_{op}}{L} \right] \frac{\pi_{op}}{L} \]

Rearrange first and second effort derivatives and efficiency wage:

\[ \frac{1}{w^* + \xi \frac{\pi_{op}}{L}} \Rightarrow \]
\[
\tilde{\eta}_{L \pi_{op} L} = \left[ -\frac{1}{\eta \tilde{e}_w} \frac{w^*}{w^* + \frac{\pi_{op}}{L}} + \frac{1}{\eta_{Q'L} \frac{\varepsilon-1}{\varepsilon} + \frac{\varepsilon-1}{\varepsilon}} \right] \frac{\xi_{\pi_{op}}}{w^* + \frac{\pi_{op}}{L}} < 0 \quad (A13)
\]

5. The Effect of Profit Sharing on Output

From Equation (24)

\[
\tilde{\eta}_{L \pi_{op} L} = \left[ -\varepsilon \frac{1}{\eta \tilde{e}_w} \frac{w^*}{w^* + \frac{\pi_{op}}{L}} + \varepsilon \frac{1}{\eta_{Q'L} \frac{\varepsilon-1}{\varepsilon} + \frac{\varepsilon-1}{\varepsilon}} \right] \frac{\xi_{\pi_{op}}}{w^* + \frac{\pi_{op}}{L}} \Rightarrow
\]

\[
\tilde{\eta}_{L \pi_{op} L} = -\varepsilon \frac{1}{\eta \tilde{e}_w} \frac{w^*}{w^* + \frac{\pi_{op}}{L}} \frac{\xi_{\pi_{op}}}{w^* + \frac{\pi_{op}}{L}} + \varepsilon \frac{1}{\eta_{Q'L} \frac{\varepsilon-1}{\varepsilon} + \frac{\varepsilon-1}{\varepsilon}} \frac{\xi_{\pi_{op}}}{w^* + \frac{\pi_{op}}{L}} \Rightarrow
\]

Insert equation (23)

\[
\tilde{\eta}_{L \pi_{op} L} = -\eta \frac{\pi_{op}}{L} \frac{w^*}{w^* + \frac{\pi_{op}}{L}} + \varepsilon \frac{1}{\eta_{Q'L} \frac{\varepsilon-1}{\varepsilon} + \frac{\varepsilon-1}{\varepsilon}} \frac{\xi_{\pi_{op}}}{w^* + \frac{\pi_{op}}{L}} \Rightarrow
\]

\[
\tilde{\eta}_{L \pi_{op} L} = \left[ -\eta \frac{\pi_{op}}{L} \frac{w^*}{w^* + \frac{\pi_{op}}{L}} + \varepsilon \frac{1}{\eta_{Q'L} \frac{\varepsilon-1}{\varepsilon} + \frac{\varepsilon-1}{\varepsilon}} \frac{\xi_{\pi_{op}}}{L} \right] \frac{1}{w^* + \frac{\pi_{op}}{L}} \Rightarrow
\]

\[
\tilde{\eta}_{L \pi_{op} L} \left[ w^* + \frac{\pi_{op}}{L} \right] = -\eta \frac{\pi_{op}}{L} w^* + \varepsilon \frac{1}{\eta_{Q'L} \frac{\varepsilon-1}{\varepsilon} + \frac{\varepsilon-1}{\varepsilon}} \frac{\xi_{\pi_{op}}}{L} \Rightarrow
\]

\[
\tilde{\eta}_{L \pi_{op} L} \left[ w^* + \frac{\pi_{op}}{L} \right] + \eta \frac{\pi_{op}}{L} w^* = \varepsilon \frac{1}{\eta_{Q'L} \frac{\varepsilon-1}{\varepsilon} + \frac{\varepsilon-1}{\varepsilon}} \frac{\xi_{\pi_{op}}}{L} \quad (a)
\]
Totally differentiate of the production function with respect variable labour costs

\[ Q = (1 + u)A \left\{ \bar{e}(w^*)L^{\frac{\frac{\varepsilon - 1}{\varepsilon}}{L}} + \left[ \bar{e} \left( \frac{\pi^{op}}{L} \right) L \right]^{\frac{\varepsilon - 1}{\varepsilon}} \right\} \Rightarrow \]

\[ dQ = (1 + u)A \frac{1}{\varepsilon - 1} \frac{F'}{F} \frac{1}{\varepsilon - 1} L^{\frac{\frac{\varepsilon - 1}{\varepsilon}}{L}} \frac{\partial}{\partial w^*} \bar{e} \left( \frac{\varepsilon - 1}{\varepsilon} \right) dL \]

\[ (1 + u)A \frac{1}{\varepsilon - 1} \frac{F'}{F} \frac{1}{\varepsilon - 1} \left( \frac{\varepsilon - 1}{\varepsilon} \bar{e} + \bar{e}' \frac{\varepsilon - 1}{\varepsilon} \right) L^{\frac{\frac{\varepsilon - 1}{\varepsilon}}{L}} \frac{\partial}{\partial L} \Rightarrow \]

\[ dQ = (1 + u)A \frac{1}{\varepsilon - 1} \frac{F'}{F} \frac{1}{\varepsilon - 1} L^{\frac{\frac{\varepsilon - 1}{\varepsilon}}{L}} \frac{\partial}{\partial L} \frac{\varepsilon - 1}{\varepsilon} \frac{\varepsilon - 1}{\varepsilon} \frac{\varepsilon - 1}{\varepsilon} dL \]

\[ (1 + u)A \frac{1}{\varepsilon - 1} \frac{F'}{F} \frac{1}{\varepsilon - 1} \left( \frac{\varepsilon - 1}{\varepsilon} \bar{e} + \bar{e}' \frac{\varepsilon - 1}{\varepsilon} \right) L^{\frac{\frac{\varepsilon - 1}{\varepsilon}}{L}} \frac{\partial}{\partial L} \Rightarrow \]

\[ \frac{dQ}{d\pi^{op}} = (1 + u)A \frac{1}{\varepsilon - 1} \frac{F'}{F} \frac{1}{\varepsilon - 1} L^{\frac{\frac{\varepsilon - 1}{\varepsilon}}{L}} \frac{\partial}{\partial \pi^{op}} + \frac{d\pi^{op}}{L} \]

\[ (1 + u)A \frac{1}{\varepsilon - 1} \frac{F'}{F} \frac{1}{\varepsilon - 1} \left( \frac{\varepsilon - 1}{\varepsilon} \bar{e} + \bar{e}' \frac{\varepsilon - 1}{\varepsilon} \right) L^{\frac{\frac{\varepsilon - 1}{\varepsilon}}{L}} \frac{\partial}{\partial \pi^{op}} \frac{dL}{d\pi^{op}} \quad (b) \]

From equation (15): \( (1 + u)A \frac{1}{\varepsilon - 1} \frac{F'}{F} \frac{1}{\varepsilon - 1} \bar{e} \left( \frac{\varepsilon - 1}{\varepsilon} \bar{e} + \bar{e}' \frac{\varepsilon - 1}{\varepsilon} \right) L^{\frac{\frac{\varepsilon - 1}{\varepsilon}}{L}} = 1 \Rightarrow (1 + u)A \frac{1}{\varepsilon - 1} \frac{F'}{F} \frac{1}{\varepsilon - 1} \bar{e} \left( \frac{\varepsilon - 1}{\varepsilon} \bar{e} + \bar{e}' \frac{\varepsilon - 1}{\varepsilon} \right) L^{\frac{\frac{\varepsilon - 1}{\varepsilon}}{L}} \]

\[ w^* + \xi \frac{\pi^{op}}{L} \]. Then, we insert in (a)

\[ \frac{dQ}{d\pi^{op}} = L \frac{d\pi^{op}}{L} L \left( w^* + \xi \frac{\pi^{op}}{L} \right) \frac{dL}{d\pi^{op}} \Rightarrow \frac{dQ}{d\pi^{op}} \]
\[ = L \frac{dw^*}{dL} + \left( w^* + \frac{\pi^o}{L} \right) \frac{dL}{dL} = \frac{dQ}{dL} \Rightarrow \]

\[ \frac{dQ}{dL} = L \left[ \frac{dw^*}{dL} + \left( w^* + \frac{\pi^o}{L} \right) \frac{dL}{L} \right] \]

Multiply both sides with \( \frac{\pi^o}{L} \) and the first part of the right side with \( \frac{w^*}{w^*} \)

\[ \frac{dQ}{dL} \frac{\pi^o}{L} Q = \frac{dw^*}{dL} \frac{\pi^o}{L} w^* + \left( w^* + \frac{\pi^o}{L} \right) \frac{dL}{L} \frac{\pi^o}{L} L = \]

\[ \frac{1}{Q} \left[ \frac{dw^*}{dL} \frac{\pi^o}{L} w^* + \left( w^* + \frac{\pi^o}{L} \right) \frac{dL}{L} \frac{\pi^o}{L} L \right] \Rightarrow \]

\[ \eta_{Q, \pi^o/L} = \frac{1}{Q} \left[ \eta_{w, \pi^o/L} w^* + \tilde{\eta}_{L, \pi^o/L} \left[ w^* + \frac{\pi^o}{L} \right] \right] \quad (c) \]

Insert (a) into (c) we have

\[ \eta_{Q, \pi^o/L} = \frac{1}{Q} \frac{1}{\eta_{Q', L} \alpha \varepsilon + \beta \varepsilon} > 0 \quad (A14) \]

6. The effect of profit sharing on profit

Taking the first order condition for profit function in respect to \( \frac{\pi^o}{L} \) give

\[ d\Pi = (1 + u)A \frac{\varepsilon}{\varepsilon - 1} F'_{\varepsilon - 1} \frac{\varepsilon - 1}{L} \frac{\pi^o}{L} \frac{\varepsilon - 1}{\pi^o} \frac{\varepsilon}{\varepsilon} dL \]

\[ \xi \frac{\partial}{\partial \frac{\pi^{op}}{L}} \frac{\pi^{op}}{L} d \frac{\pi^{op}}{L} L \Rightarrow \]

\[ d\Pi = (1 + u)A \frac{\varepsilon}{\varepsilon - 1} F'_{\varepsilon - 1} L^{-\frac{\varepsilon - 1}{\varepsilon}} \frac{\varepsilon - 1}{\varepsilon} \hat{e}^{\frac{\varepsilon - 1}{\varepsilon}} d \frac{\pi^{op}}{L} - \xi L d \frac{\pi^{op}}{L} \Rightarrow \]

\[ \frac{d\pi}{\pi^{op}} \frac{\pi^{op}}{L} = (1 + u)A \frac{1}{\varepsilon} F'_{\varepsilon - 1} \hat{e}^{\frac{1}{\varepsilon}} L^{-\frac{1}{\varepsilon}} \frac{\varepsilon - 1}{\varepsilon} - \xi L \Rightarrow \]

\[ \frac{d\pi}{\pi^{op}} \frac{\pi^{op}}{L} = \left[ (1 + u)A \frac{1}{\varepsilon} F'_{\varepsilon - 1} \hat{e}^{\frac{1}{\varepsilon}} L^{-\frac{1}{\varepsilon}} \frac{\varepsilon - 1}{\varepsilon} - \xi L \right] \frac{\pi^{op}}{L} - \xi L \Rightarrow \]

\[ \left[ (1 + u)A \frac{1}{\varepsilon} F'_{\varepsilon - 1} \hat{e}^{\frac{1}{\varepsilon}} L^{-\frac{1}{\varepsilon}} - \xi \right] \frac{\pi^{op}}{\Pi} L > 0 \quad (A15) \]