A Two-production-period Model with State-owned and Labour-managed Firms

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Abstract: This paper considers a two-production-period model in which a state-owned firm competes against a labour-managed firm. In the first production period, the state-owned and labour-managed firms simultaneously and independently choose outputs. The chosen outputs become common knowledge and then, in the second production period, the firms simultaneously and independently choose outputs. After the second period outputs have been chosen, the market opens. The paper shows that there exists a subgame perfect Nash equilibrium that coincides with the Stackelberg outcome in which the labour-managed firm is the leader. Therefore, we find that in equilibrium the state-owned firm cannot play the role of the Stackelberg leader, whereas the labour-managed firm can.

Keywords: labour-managed firm, mixed duopoly, quantity-setting competition, state-owned firm, two production periods

JEL classifications: C72, D21, L30

1. Introduction

This paper studies the behaviours of a state-owned welfare-maximising firm and a labour-managed income-per-worker-maximising firm in a Cournot model. As is very well known, state-owned public firms exist in developed and developing countries as well as in former communist countries, and compete with private firms in many industries.

The first work on a theoretical model of a state-owned firm was conducted by Merrill and Schneider (1966). Thereafter, many economists have analysed mixed market models that incorporate state-owned firms. Most of these studies consider quantity-setting models with homogeneous goods and assume that state-owned public firms are less efficient than private firms or the marginal costs of public and private firms are increasing. Nett (1991, 1994), Delbono
and Denicolò (1993) and Poyago-Theotoky (1998) investigate mixed models with R&D. For example, Poyago-Theotoky (1998) shows that, first, in the mixed duopoly a public firm invests more in R&D than a private firm, second, in the mixed duopoly the private firm reduces its R&D investment relative to the private duopoly while the public firm spends relatively more on R&D, and third, relative to the social optimum, the public firm overinvests while the private firm underinvests in R&D in the mixed duopoly.

Ware (1986), Willner (1994), Wen and Sasaki (2001), Nishimori and Ogawa (2004) and Lu and Poddar (2009) construct mixed models in which firms choose capacity. Lu and Poddar (2009) consider a model of endogenous timing of sequential choice of capacity and quantity with observable delay in a mixed duopoly and show that a simultaneous play at the capacity stage or at the quantity stage can never be supported as subgame perfect Nash equilibrium.

White (1996), Poyago-Theotoky (2001), Myles (2002), Fjell and Heywood (2004) and Kato and Tomaru (2007) investigate the relationship between the output subsidy in mixed oligopoly and that in private oligopoly. Poyago-Theotoky (2001) shows that the optimal output subsidy is identical and profits, output and welfare are also identical irrespective of whether the state-owned public firm moves simultaneously with private firms or it acts as a Stackelberg leader or all firms behave as profit-maximisers. Pal and White (1998) study the effects of privatisation in the presence of strategic trade policies within an international mixed oligopoly serving a single market and show that if subsidies are used, then privatisation always lowers the subgame perfect Nash equilibrium level of subsidy.

Pal (1998) studies the subgame perfect Nash equilibrium of a mixed market, where firms first choose the timing for selecting their quantities and shows that behaving like a private firm is not the optimal role for a public firm. Matsumura and Matsushima (2003) investigate the sequential choice of location in a mixed duopoly, where a public firm competes against a private firm, and consider the effect of price regulation. They show that the public firm should become the follower (leader) if a price regulation is (is not) imposed.

Quite a few economists study price-setting competition with homogeneous goods or differentiated goods. For example, Ogawa and Kato (2006) examine price competition in a homogenous product market under a mixed duopoly and show that the equilibrium price in the private price leadership case is higher than the one in the simultaneous case under some cost conditions and always exceeds the one in the public price leadership case. Bárcena-Ruiz and Garzón (2007) examine the issue of capacity chosen by firms in a mixed duopoly, by considering that firms compete on prices and show that the public firm chooses over-capacity when products are substitutes and under-capacity when products are complements.
There are many further studies such as Cremer et al. (1991), George and la Manna (1996), Fjell and Pal (1996), Garvie and Ware (1996), Anderson et al. (1997), Mujumdar and Pal (1998), Bárcena-Ruiz and Garzón (2003), Matsumura and Kanda (2005), Nishimori and Ogawa (2005), Ohnishi (2006), Bárcena-Ruiz (2007) and Fernández-Ruiz (2009). However, these studies consider mixed market models in which state-owned firms compete against profit-maximising capitalist firms, and do not include labour-managed firms.

Labour-managed firms have existed in Western economies since the advent of the factory system. The oldest surviving labour-managed firms in the United Kingdom and Italy appeared in the nineteenth century (Bonin et al. 1993). After the Second World War, the right to manage the firm in the former Yugoslavia was, within the limits determined by law, in the hands of its employees (Furubotn and Pejovich, 1970). The labour-managed firm in all Western European countries grew significantly between the early 1970s and the early 1980s, for example, from 4,370 firms in 1970 to 11,203 in 1982 in Italy and from 522 to 933 firms in France over the same period. Furthermore, in the United Kingdom the number of labour-managed firms rose by almost 1,000 per cent and employment by 133 per cent between 1976 and 1981 (Estrin, 1985). In the United States, the most notable examples of labour-managed firms are in the plywood industry in the Pacific Northwest where they have been in existence since 1921, and during the 1950s, they contributed as much as 25 per cent of the industry’s total output (Bonin et al., 1993). Furthermore, in China, the market-oriented economic reform has given much greater autonomy to state and collective enterprises’ managers to make production, investment and marketing decisions. Meng and Perkins (1998) find that the state and the collective sectors behave more like labour-managed firms in that they try to maximise income per worker rather than profit, whereas private-sector firms are profit maximisers.

The pioneering work on a theoretical model of a labour-managed firm was done by Ward (1958). Since then, mixed market models that incorporate labour-managed firms have been studied by many economists. So far, the literature on labour-managed firms has mainly dealt with quantity-setting competition with homogeneous goods, disregarding price-setting behaviour. In Mai and Hwang (1989), Horowitz (1991), Okuguchi (1991), and Sakai (1993), the labour-managed firm and the profit-maximising firm each decide only how much output to produce or how much labour to employ. Cremer and Crémer (1992) extend their analyses to the case in which the firms decide both the employment level and the capital stock simultaneously and show that the labour-managed firm produces less output than the profit-maximising firm in a two-stage Cournot duopoly regime. However, Futagami and Okamura (1996) examine a three-stage duopoly model in which a labour-managed firm
and a profit-maximising firm use investments as strategic variables, and show that the labour-managed firm invests more capital and produces more than the profit-maximising firm. In addition, Lambertini and Rossini (1998) examine the behaviour of labour-managed and profit-maximising firms in a Cournot duopoly with capital strategic interaction and show that the labour-managed firm tends to over-invest while the opposite holds for the profit-maximising firm irrespective of the capital rental price.

Lambertini (1997) considers a mixed duopoly where a profit-maximising and a labour-managed firm compete either in prices or in quantities and shows that if firms can choose the timing of moves before competing in the relevant market variable, the Bertrand game yields multiple equilibria, while the Cournot game has a unique subgame perfect equilibrium with the profit-maximising firm in the leader’s role and the labour-managed firm in the follower’s role. Lambertini (2001) investigates the nature of the equilibria arising under spatial differentiation in a duopoly model where at least one firm maximises value added per worker and shows that if firms’ objectives differ, there exists a subgame perfect equilibrium in pure strategies, which is possibly characterised by asymmetric locations. Ireland (2003) examines a price-setting mixed model in which consumer information is imperfect and shows that the average prices of all firms increase with the number of labour-managed firms, but labour-managed firms price lower than profit-maximising firms. There are many further studies such as Stewart (1991, 1992), Askildsen and Ireland (1993), Ireland and Stewart (1995), Neary and Ulph (1997), Ohnishi (2008) and Cuccia and Cellini (2009). However, these studies do not include state-owned firms.

In the real world, we can find lots and lots of profit-maximising capitalist firms. As is stated above, we can also find lots of real world examples of labour-managed firms. Some studies examine mixed market competition with state-owned and labour-managed firms. Delbono and Rossini (1992) explore the creation of (i) a duopoly formed by a labour-managed firm and a state-owned firm in a Cournot-Nash setting, and (ii) a horizontal merger between the same agents. In addition, Ohnishi (2009) investigates the behaviours of a state-owned firm and a labour-managed firm in a two-stage mixed market model with capacity investment as a strategic instrument. However, there are few studies that examine mixed market models with state-owned and labour-managed firms.

Therefore, we consider a mixed Cournot model with two production periods in which state-owned and labour-managed firms compete with each other. Saloner (1987) examines a pure Cournot duopoly model with two production periods in which profit-maximising firms compete against each other and shows that any outcome on the outer envelope of the best response
functions between and including the firms’ smallest Stackelberg outcomes, is sustainable as a subgame perfect Nash equilibrium. In addition, Matsumura (2003a) adopts Saloner’s (1987) duopoly model with two production periods and investigates endogenous roles in a mixed duopoly where public and private firms compete. He finds that in equilibrium the public firm cannot play the role of the Stackelberg leader, while the private firm can.

We investigate endogenous roles in two-production-period Cournot competition in which a state-owned welfare-maximising firm competes with a labour-managed income-per-worker-maximising firm. We present the subgame perfect Nash equilibrium of the mixed Cournot model with two production periods before the market clears.

The remainder of this paper is organised as follows. In Section 2, we describe the mixed duopoly model. Section 3 presents the equilibrium of the mixed duopoly model. Finally, Section 4 concludes the paper.

2. The Model

We deal with a mixed duopoly, where the state-owned welfare-maximising firm is designed as firm 1 and the labour-managed income-per-worker-maximising firm as firm 2. Both firms produce perfectly substitutable goods. In the remainder of this paper, superscripts 1 and 2 refer to firms 1 and 2, respectively, and subscripts 1 and 2 refer to periods 1 and 2, respectively. In addition, when $i$ and $j$ are used to refer to firms in an expression, they should be understood to refer to 1 and 2 with $i \neq j$. The market price is determined by the inverse demand function $P(X)$, where $X = x^1 + x^2$. We assume that $P' < 0$ and $P'' \geq 0$.

The timing of the game is as follows. In the first production period, firms 1 and 2 simultaneously and non-cooperatively choose outputs $x^1_1 \geq 0$ and $x^2_1 \geq 0$, respectively. Each firm knows $x^1_1 \geq 0$ and $x^2_1 \geq 0$ and then, in the second production period, the firms simultaneously and non-cooperatively choose outputs $x^1_2 \geq 0$ and $x^2_2 \geq 0$. After the second period outputs have been chosen, price is determined from the inverse demand function $P(x^1_1 + x^1_2 + x^2_1 + x^2_2)$, and the firms sell cumulative outputs $x^1 \equiv x^1_1 + x^1_2$ and $x^2 \equiv x^2_1 + x^2_2$.

Therefore, social welfare, which is the sum of consumers’ surplus and total profits by the firms, is given by

$$W = \int_0^X P(q) dq - r(x^1) - w(x^1) - r(x^2) - w(x^2) - 2f,$$

where $r$ denotes the capacity (capital) cost function, $w$ the labour cost function, and $f > 0$ the fixed cost.
Firm 2’s income per worker is given by

$$\nu^2 = \frac{P(X)x^2 - r(x^2) - f}{l(x^2)},$$  \hfill (2)

where $l$ denotes the labour input function.\textsuperscript{5} We assume that $l' > 0$ and $l'' > 0$. This assumption means that the marginal labour input is increasing. Furthermore, we assume $r' > 0$, $r'' > 0$, $w' > 0$ and $w'' > 0$.\textsuperscript{6}

Throughout this paper, we use subgame perfection as our equilibrium concept.\textsuperscript{7} Since inverse demand is defined only for non-negative outputs, it is ensured that all outputs obtained in equilibrium are non-negative.

Firm 1’s reaction function is defined by

$$R^1(x^2) = \arg \max_{x^1 \geq 0} \left[ \int_0^{x^1} P(q) dq - r(x^1) - w(x^1) - r(x^2) - w(x^2) - 2f \right].$$  \hfill (3)

The equilibrium occurs where each firm maximises its objective function with respect to its own output level, given the output level of its rival. That is, firm 1 aims to maximise (1) with respect to its own output level, given the output level of firm 2. The first-order condition for firm 1 is

$$P - r' - w' = 0,$$  \hfill (4)

and the second-order condition is

$$P' - r'' - w'' < 0.$$  \hfill (5)

Furthermore, we have

$$R^1(x^2) = -\frac{P'}{P' - r'' - w''}.$$  \hfill (6)

Since $P' < 0$, $R^1(x^2)$ is downward sloping.\textsuperscript{8}

Firm 2’s reaction function is defined by

$$R^2(x^1) = \arg \max_{x^2 \geq 0} \left[ \frac{P(X)x^2 - r(x^2) - f}{l(x^2)} \right].$$  \hfill (7)

Firm 2 aims to maximise (2) with respect to its own output level, given the output level of firm 1. The first-order condition for firm 2 is
\[(P'x^2 + P - r')l - (Px^2 - r - f)'l' = 0, \quad (8)\]

and the second-order condition is
\[(P''x^2 + 2P' - r'')l - (Px^2 - r - f)l'' < 0. \quad (9)\]

Furthermore, we have
\[R^2(x^1) = -\frac{P''x^2l + P'(l - x^2l')}{(P''x^2 + 2P' - r'')l - (Px^2 - r - f)l''}. \quad (10)\]

Since \(l'' > 0, l - x^2l' < 0\), so that \(P''x^2l + P'(l - x^2l')\) is positive; that is, \(R^2(x^1)\) is upward sloping.\(^9\)

These reaction functions ensure that there exists a unique single-production-period Cournot-Nash equilibrium, which is denoted by \((N^1, N^2)\).

3. Equilibrium

In this section, we begin by defining firm \(i\)’s Stackelberg leader output. Firm \(i\) selects \(x^i\), and firm \(j\) selects \(x^j\) after observing \(x^i\). The Stackelberg equilibrium is denoted by \((L', F')\), where \(L'\) is the leader’s output and \(F'\) is the follower’s.

We now state the following lemma.

Lemma 1. \(L' > N'\).

Proof. First, we prove that firm 1’s Stackelberg leader output is higher than its Cournot output. Firm 1 maximises \(W(x^1, R^2(x^1))\) with respect to \(x^1\). Therefore, firm 1’s Stackelberg leader output satisfies the first-order condition:

\[\frac{\partial W}{\partial x^1} + \frac{\partial W}{\partial x^2} \frac{\partial R^2}{\partial x^1} = 0, \quad (11)\]

where \(\partial W/\partial x^2\) is positive, and \(\partial R^2/\partial x^1\) is also positive. To satisfy (11), \(\partial W/\partial x^1\) must be negative.

Second, we prove that firm 2’s Stackelberg leader output is higher than its Cournot output. Firm 2 maximises \(W^2(x^2, R^1(x^2))\) with respect to \(x^2\).
Therefore, firm 2’s Stackelberg leader output satisfies the first-order condition:

$$\frac{\partial V^2}{\partial x^2} + \frac{\partial V^2}{\partial x^1} \frac{\partial R^1}{\partial x^2} = 0,$$

where $\partial V^2 / \partial x^1 = P'x^2$ is negative from $P' < 0$, and $\partial R^1 / \partial x^2$ is also negative. To satisfy (12), $\partial V^2 / \partial x^2$ must be negative. Thus, the lemma follows. Q.E.D.

Each firm chooses its own total output under the constraint that the final output is not strictly smaller than its first-period production. The following lemma states how the first-period production affects the outcome in the second period.

**Lemma 2.**

(i) If $(x'_i, x'_j) \leq (N^i, N^j)$, then $(x'_i, x'_j) = (N^i, N^j)$.

(ii) If $x'_i \geq N^i$, then $x'_2 = 0$.

(iii) If $x'_i > N^i$ and $x'_j < N^j$, then $x'_i = \max\{R^i(x'_i), x'_j\} < N^j$.

Since firm $j$ determines its output before observing $x'_2$, firm $i$’s second-period production $x'_2$ has no strategic value. Therefore, if firm $i$’s first-period output is lower than its Cournot output, then it has no incentive to produce a larger output than its Cournot output in the second period. Hence, the equilibrium in the second period is at $(x'_i, x'_j) = (N^i, N^j)$ (Lemma 2 (i)). There is no depreciation on $x'_i$. If $(x'_i, x'_j) > (N^i, N^j)$, then $(N^i, N^j)$ is not an equilibrium. If firm $i$ chooses $x'_i > N^i$, then it does not produce any additional output (Lemma 2 (ii)). Given that $x'_i > N^i$, firm $i$’s total output equals $x'_i$. Therefore, firm $j$ chooses its total output $x^j = R^j(x'_i)$ as long as $R^j(x'_i) \leq x'_i$ (Lemma 2 (iii)). Romano and Yildirim (2001) prove that these hold true under general payoff functions.

We now discuss the equilibrium of the mixed duopoly model. The main result of this study is described by the following proposition.

**Proposition 1.** In the mixed duopoly model, there exists a subgame perfect Nash equilibrium that occurs at $(F^1, L^2)$. 

Proof. From Lemma 2, firm $i$’s output is as follows:

$$
x_i^i = \begin{cases} 
  x_i^i & \text{if } x_i^i \geq N^i \\
  \max \{x_i^i, R_i(x_i^i)\} & \text{if } x_i^i \geq N^j \\
  N_i & \text{if } x_i^i < N^j \text{ and } x_i^j < N^j.
\end{cases} \quad (13)
$$

We consider the optimal output of each firm. Lemma 1 states that firm 1’s Stakelberg leader output is higher than its Cournot output. Let $W$ be assumed to be continuous and concave in $x^1$. Hence, firm 1 prefers $x^1 > N^1$. If social welfare is higher with firm 1’s Stakelberg leader output than with its Stakelberg follower output, then it wants to choose its Stakelberg leader output. On the other hand, if social welfare is higher with firm 1’s Stakelberg follower output than with its Stakelberg leader output, then it wants to choose its Stakelberg follower output. Because cycling of choices is impossible, the equilibrium is either $(L^1, F^2)$ or $(F^1, L^2)$.

Lemma 1 shows that firm 2’s Stakelberg leader output is higher than its Cournot output. Let $V$ be assumed to be continuous and concave in $x^2$. The further a point on $R^2$ gets from firm 2’s Stackelberg leader point, the more its income per worker decreases. Hence, firm 2 prefers $x^2 > N^2$ above all others, and the equilibrium becomes $(F^1, L^2)$.

From (13), we see that the equilibrium outcome is decided by the value of $x_i^i$. Our equilibrium concept is the subgame perfect equilibrium and all information in the model is common knowledge. Firm 2 chooses $x_i^2$ associated with its Stackelberg leader solution. Thus, the proposition follows. Q.E.D.

We now discuss the role of backward induction in firm 1’s behaviour. Our equilibrium concept is subgame perfection and all information in the model is common knowledge. If firm 2 plays the role of the Stackelberg leader, then social welfare is improved, so that firm 1 becomes the Stackelberg follower. Firm 2’s Stackelberg leader output is larger than its Cournot output, and firm 1’s Stackelberg follower output is smaller than its Cournot output. Firm 1 knows that the sum of outputs of two production periods must be its Stackelberg follower output. Each firm chooses its own total output under the constraint that the final output is not strictly smaller than its first-period production. Lemma 2 shows
the outcomes in the second period given $x_1^1$ and $x_1^2$. In consideration of these, firm 1 chooses $x_1^{1*}$, where $x_1^{1*} \leq F^1 < N^1$.

4. Concluding Remarks

We have examined a mixed Cournot duopoly model with two production periods in which a state-owned welfare-maximising firm competes with a labour-managed income-per-worker-maximising firm, and have shown that there exists a subgame perfect Nash equilibrium that coincides with the Stackelberg outcome in which the labour-managed firm is the leader. Therefore, we have found that in equilibrium the state-owned firm cannot play the role of the Stackelberg leader, whereas the labour-managed firm can.

On the other hand, the following studies consider mixed Cournot models and show that state-owned firms can become the Stackelberg leader. Fjell and Heywood (2002) consider a mixed model, in which a state-owned public firm competes as a Stackelberg leader with domestic and foreign profit-maximising private firms, and show that regardless of the mix of foreign and domestic firms, the public leader always produces less than under a Cournot conjecture. In addition, Matsumura (2003b) investigates a Stackelberg mixed duopoly where a domestic public firm competes against a foreign profit-maximising private firm. He shows first that the public firm chooses its output and that the equilibrium coincides with the Stackelberg solution where the public firm is the leader.

Finally, we provide policy implications for the organisation of public sectors. Our results indicate that the state-owned firm behaves as a follower, i.e., the state-owned firm becomes a less aggressive competitor against the domestic labour-managed firm. If the state-owned firm behaves less aggressively, then perceiving this fact, the labour-managed firm has an incentive to be more aggressive. The labour-managed firm’s income per worker is highest at its Stackelberg leader point on the state-owned firm’s reaction function. Therefore, the labour-managed firm becomes the Stackelberg leader and the state-owned firm plays the role of the Stackelberg follower. The labour-managed firm’s output exceeds its Cournot output, and thus it increases its output. Increasing the labour-managed firm’s output increases total market output, thereby improving consumer surplus. In addition, increasing its output and income per worker increases producer surplus. Hence, more aggressive behaviour by the labour-managed firm improves social welfare. Therefore, we see that governments that wish to increase social welfare should adopt industrial policies that make state-owned firms behave less aggressively toward labour-managed firms and increase the outputs of labour-managed firms.
Notes

3 See, for example, Okuguchi (1991), Cremer and Crémer (1992), Lambertini (1997) and Ireland (2003) for Bertrand competition with substitute goods.
4 This paper is an extension of the works by Saloner (1987) and Matsumura (2003a) and thus the model in the paper is basically the same as their models except for types of firms.
5 Profit-maximising capitalist firms’ profits are distributed to shareholders in the form of dividends, whereas those of labour-managed firms are distributed to members.
6 We assume that both firms share the same cost function and the marginal cost is increasing. This assumption is often used in literature studying mixed markets. See, for instance, Harris and Wiens (1980), Ware (1986), Delbono and Rossini (1992), Fjell and Pal (1996), White (1996), Pal and White (1998), Poyago-Theotoky (1998), Fjell and Heywood (2002), Bárcena-Ruiz and Garzón (2003) and Matsumura and Kanda (2005). If the marginal cost is constant or decreasing, then firm 1 produces an output such that its price equals its marginal cost and supplies the entire market, resulting in a welfare-maximising public monopoly. This assumption is made to eliminate such a trivial solution.
7 This paper presents the subgame perfect Nash equilibrium of perfect information in the same way as Saloner (1987) and Matsumura (2003a). The paper describes precisely the equilibrium by using perfect information.
8 For the reaction functions of state-owned firms, see Nett (1993), Matsumura (2003a) and Ohnishi (2006).
9 For the reaction functions of labour-managed firms, see Stewart (1991), Delbono and Rossini (1992), Lambertini and Rossini (1998) and Ohnishi (2008). It is very well known that the reaction function of profit-maximising capitalist firms is downward sloping in Cournot games. On the other hand, the reaction function of labour-managed firms is generally upward sloping.

References


