Factors Affecting Differential Equation Problem Solving Ability Of Students At Pre-University Level: A Conceptual Model

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ABSTRACT

In this study, different factors affecting students' differential equations (DEs) solving abilities were explored at pre university level. To explore main factors affecting students' differential equations problem solving ability, articles for a 19-year period, from 1996 to 2015, were critically reviewed and analyzed. It was revealed that combination of four factors; "epistemological math problem solving beliefs, belief about usefulness of mathematics, self-regulated learning (SRL) strategies and goal orientations" have great potential to enhance differential equation problem ability. Based on these findings, a conceptual model was developed and presented in this paper to enhance differential equation problem solving ability. This study has provided several important implications for the curriculum designers and teachers to enhance conceptual understanding in differential equation problem solving, particularly in the developing countries..

Keywords: *Algebraic Approach, Context Familiarity, Differential Equation*

INTRODUCTION

Differential equations (DEs) play an important role in mathematics and remained an essential course in calculus for centuries. The concept of DEs is used to model and understand real life problems. These provide opportunities to formulate the application of phenomena from other discipline of science and social science fields such as Physics, Astronomy, Biology, and Economics (Arslan, 2010). Therefore, it has been integrated in diverse courses of several departments including college level (Blumenfeld, 2006).

Several methods have been implemented and more are being developed to make the differential equations (DEs) problem solving easy, interesting and productive for students. A variety of studies were conducted to identify major contribution factors to solve DE problems. Literature reveals that there are three major cognitive and contributing factors, including knowledge, control (metacognition) and beliefs, which enable students to solve mathematics problem and also to overcome difficulties (Kroll et al., 1993). Among these factors, beliefs are the most essential components to generate meaning and build up overall intention that define the context for learning mathematics (Cobb, 1986).

Mathematics educators also have the same opinion that formal mathematics education has a significant influence on the development of student's mathematics beliefs. However, social or cultural processes are also important while considering students mathematical growth (Cobb et al., 1995; Lave, 1988; Rogoff, 1990). Therefore, the primary focus of mathematics educators was to explore the individual psychological aspects of learning undergraduate mathematics (Harel et al., 1998; Tall et al., 1981). In this context, Yackel et al. (2000) well supported the mathematics educators and suggested that students beliefs about their own role as well as others role, and the general nature of mathematical activity and the classroom social norms all are equally involved. Author analyzed social interaction patterns, social and socio mathematical norms, to explore the effect of these norms towards differential equations problem solving (Yackel et al., 2000). Similar observations were revealed in few other studies. It was concluded that students emerging beliefs concerned with the capability to generate mathematics ideas and the role of reasoning and justification are automatically connected with the social and socio-mathematical norms of their classroom settings (Yackel et al., 2002).

In similar context, Ju et al. (2007) documented the transformation of students mathematics beliefs especially for the case of differential equation, regarding to their relation with mathematics, and their role in the classroom practice. Discourse analysis showed that student's portray a shift from third person to first person perception in such manner; presume change in student's beliefs. Consequently, transformation of students' beliefs depends on classroom learning environment, including their own cognitive assists, role of teacher and their teaching resources.

Recently, several other researchers also observed a strong relationship between beliefs about mathematics and mathematical performance (Beghetto et al., 2012; Schommer- Aikins et al., 2005; Schommer-Aikins et al., 2013). Therefore, focusing on the students' beliefs, associated with science and especially math problem solving remained a highly promising area of investigation. Likewise, McLeod (1992) have same opinion that mathematics beliefs enhance or weaken individual's mathematical and problem solving ability. These beliefs further effect students learning approaches. Several other researchers introduced contemporary self-regulated learning (SRL) theory and explored the role of epistemological beliefs in mathematical problem solving (Hofer, 1999; Muis, 2004, 2008; Stockton, 2010). Epistemological beliefs influence students learning approaches resulting into better mathematics achievement. Numerous studies also correlated the implication of students' self-regulated learning skills with goals and goal orientation beliefs (Pintrich, 1991). Muis (2007) further interlinked epistemological beliefs, goal orientation, learning strategies, and math achievement. In addition to these constructs, Schommer (2013) also reported that the belief about usefulness of mathematics strongly affects math problem solving.

To explore various factors affecting students' differential equations problem solving ability, articles for a 19-year period, from 1996 to 2015, were critically reviewed. It was concluded that four factors "epistemological math problem solving beliefs, usefulness, self-regulated learning strategies (SRL) and goal orientations have great potential to solve differential equation problem (Aisha et al., 2017). Although this recently accepted review article has well described different factors and methodologies to solve differential equation problem pre university level and also predicted the potential areas for the future research. However, the previously highlighted four factors "epistemological math problem solving beliefs, usefulness, self-regulated learning strategies (SRL) and goal orientations were required to be elaborated in detail. Therefore, this study was conducted to explore these factors in detail. In addition, these factors were organized to develop a conceptual model for differential equations problem solving for the pre university level students.

Research Objectives

The aim of current article is to determine how different factors affect students' differential equations problem solving. It is anticipated that the findings of current study will assist educators and researchers with some insightful ideas about the pattern and issues studied in the area of differential equations.

The research questions addressed by this study are therefore:

1. What are the main factors, affecting students' differential equations problem solving?
2. How these different factors can be organized to develop a conceptual model for differential equations problem solving?

METHODOLOGY/RESEARCH DESIGN

To explore main factors affecting students' differential equations problem solving ability, articles for a 19-year period, from 1996 to 2015, were critically reviewed and analyzed (Aisha et al., 2017). Based on this review article, it was revealed that the four factors; epistemological math problem solving beliefs, belief about usefulness of mathematics, self-regulated learning (SRL) strategies and goal orientations have great potential to enhance differential equation problem ability. These factors were further critically studied and a conceptual model was developed and presented in this paper for effective differential equation problem solving. Details of these factors have been described in the following sections.

FACTORS' IDENTIFICATION

Mathematical problem solving plays a pivotal role in students learning of mathematics. To assure success in solving problems, knowing appropriate algorithms, facts, and procedures are not sufficient. Instead, there are some others factors which depend on more than the prerequisite mathematical content knowledge and also, influence direction as well as performance outcome. These factors include employment of different learning strategies, the emotions (like anxiety, frustration, enjoyment), and the beliefs about mathematical tasks (Garofalo, 1989; Schoenfeld, 1985a). Among these factors, beliefs are the most essential components as they became reason of learning mathematics and generally create motivations that define the context for learning mathematics (Cobb, 1986). Beliefs are further classified as domain general labeled as epistemological beliefs and domain specific known as epistemological mathematical problem solving beliefs including usefulness (part of domain specific belief). Several studies well confirmed that both types of beliefs influence many aspects of cognitive and also their math problem solving performance (Schommer-Aikins et al., 2005; Schommer-Aikins et al., 2013; Schommer-Aikins et al., 2002). Because, epistemological beliefs effect students learning strategies/approaches and consequently their learning outcomes (Schommer, 1990). Recently, Abedalaziz et al. (2012) investigated student's epistemological mathematical problem solving beliefs and academic achievement in the context of Malaysia. The findings from multiple regression revealed that the five scales are able to predict mathematical achievement significantly. However, the strongest predictor was belief regarding to the role of effort in increasing mathematical ability because, students were convinced and motivated enough to do their best.

Several other researchers extended their studies to analyze the relationships between beliefs and SRL. Findings show that SRL processing and epistemological beliefs are interrelated constructs (Bråten et al., 2005; Hofer et al., 1997; Muis et al., 2009; Schommer-Aikins, 2004). Schommer-Aikins (2004) hypothesized reciprocal relationship between epistemological beliefs and SRL, whereas, experimental results shown the existence of this relationship in multiple contexts (Bråten et al., 2005; Hofer, 1999; Muis, 2008). Later on, a few researchers infused these two constructs into the study of mathematical problem solving (Hofer, 1999; Muis, 2004, 2008; Stockton, 2010). Findings revealed that student learning is located within the interrelated constructs of epistemological beliefs, self-regulated learning and problem solving. Typically, successful problem solvers have a control over the problem space, and also have sophisticated epistemological beliefs (Muis, 2008; Perels et al., 2005; Schoenfeld, 1983, 1985b, 1989). Beside this, the role of goal orientation and self-regulated learning were also remained most promising area in analyzing mathematical problem solving skills (P. R. Pintrich, 1991).

Goal orientations are a part of self-motivational beliefs (Zimmerman et al., 2000). These beliefs work as an stimulating agent for an individual's self-regulatory behavior and effect the execution of self-regulatory knowledge and skills (Kingir et al., 2013; Montalvo et al., 2004). Students' goal orientations are further categorized as mastery, performance and avoidance goals (Ames, 1992; C. Kadioglu et al., 2014). Mastery goal oriented students are more likely to use deeper level of cognitive strategies such as elaboration and organizational strategies (P. R. Pintrich et al., 1992).

In addition, mastery goal is positively related to metacognitive as well as self-regulatory strategies, for instance planning, monitoring, and regulating learning (García et al., 1991; P. R. Pintrich et al., 1990; P. R. Pintrich et al., 1994). In spite of several successful findings, there were some inconsistencies in the literature concerned about the role of goal orientations.

Recently, Fadlelmula et al. (2015) examined the interrelationship among goal orientation, use of self-regulated strategies and mathematics' achievement. Findings showed that only mastery goal was related to SRL strategies and math achievement. Among SRL, only elaboration was significant predictor of math achievement. These findings were partially supporting the previous studies, wherein both mastery and performance goals were positive predictors of self-regulated learning strategies and generated adaptive outcomes (Liem et al., 2008).

Most of the researchers who had investigated the trichotomous goal frame work reported that mastery and performance-oriented learners have shown more tendency towards self-regulation than avoidance goal one (He, 2004). These researchers further argued that students adopting both mastery and performance goal orientations can accomplish important guides for interpreting feedback and regulate their learning (Butler et al., 1995; García et al., 1991). As contrary to mastery and performance goal, achievement goal theory proposed that avoidance goal is basically based on negative beliefs (i.e. fear of failure or rejection). Therefore, avoidance goal oriented students mostly give up when they face difficult and tedious task (Liem et al., 2008). Many studies reported that avoidance goal has negative effect on math achievement (Elliot et al., 2001; Elliot et al., 1999; Wolters, 2004). Rastegar (2006), and Hejazi, et al.'s (2008) observed similar indirect effects of avoidance goal on mathematics achievement via cognitive strategies.

Domain difference may deviate student's perception which subsequently influence the relationship between goal orientation and self-regulated learning (Grossman et al., 1995). To elaborate these facts, Wolters et al. (1996) studied this relationship and replicate findings across three academic subjects, Math, English and Social study. Afterward, same scheme was used for chemistry course (C. Kadioglu et al., 2014; C. Kadioglu et al., 2011; C. Kadioglu, Uzuntiryaki E., & Capa-Aydin Y., 2009). Findings have illustrated that both master and performance approach goal significantly predicted students SRL.

Muis (2007) prolonged these constructs and theoretically interlinked the epistemological beliefs, goal orientation, learning strategies, and achievement. Same group of authors had used empirically test to examine these factors (Muis et al., 2009). Findings revealed that epistemological beliefs influenced the adopted goals, which in turn affects their learning strategies they used in their achievement. In addition, goal orientations have shown the mediating role between epistemological beliefs and learning strategies. Similarly, learning strategies also mediate the relationship between goal orientations and achievement.

An another remarkable effort was noticed by Rastegar et al. (2010), who had investigated the mediating role of goal orientation, mathematics self-efficacy, and cognitive engagement between epistemological beliefs and math achievement. Findings clearly confirmed that these three constructs mediate this relationship.

CONCEPTUAL MODEL

Overall literature reveals that epistemological beliefs, usefulness, goal orientation, and self-regulated learning strategies have significant role in mathematics achievement as well as problem solving. However, researcher could not able to see any study, showing the combined effect of these four actors towards mathematics problem solving. In addition, it may be hypothesized that if afore cited positively affect the mathematics problem solving, then it can also effect the differential equation based problem solving. Therefore, in the present study, major contribution of these four factors, epistemological belief, usefulness, goal orientation and self-regulated learning strategies on the differential equation problem solving were discussed. A detail conceptual frame work was proposed and major contributing factors were organized in a productive mode (Figure 1).

The first construct "epistemological math problem solving beliefs" may be measured with an adapted scale taken from Indiana mathematics beliefs scale (IMBS; (Kloosterman et al., 1992). Indiana mathematics belief scale (IMBS) measures student's beliefs about mathematics problem solving. This scale is usually evaluated by the following five dimensions; 1) duration of problem, 2) steps, 3) word problems, 5) effort.

Duration of problem comprised of six items, such as, "Differential equation word problems that take long time don't disturb me". The measure of steps composed of six items, such as, "Differential equation word problems can be solved without remembering formula." The measure of understanding included six items, such as, "Time used to. examine why a solution works is considerably not time passed". In addition, the measure of word problems consisted of six items, such as, "A student who can't solve word problems really can't understand and solve differential equations". Similarly, effort scale also comprised of six items, such as, "Practice can improve one's ability to solve differential equations

The second construct "Usefulness" may be measured by adapting the Fennema et al. (1976) scale. Usefulness scale include six items, such as, "Differential equation problems are worthy and compulsory".

The third construct "goal orientations" is generally classified into three types, including mastery, performance and avoidance goal. Mastery goal orientation also labeled as task or learning goal orientation. Across these different labels, basic construct is same with minor theoretical differences attached to them. However, in mastery goal orientations focus of the students is on mastery of the subject matter. Whereas, performance goal orientations often known as ability or ego orientation, where students are provoked to show their performance as compared to the other students (Ames, 1992; Anderman et al., 1994; Dweck et al., 1988; Nicholls, 1984; P. Pintrich et al., 1996; Urdan et al., 1995). The nature of both mastery goal and performance goal is different from each other. Due to dissimilar nature, both these beliefs influence outcomes differently. In contrast, avoidance goal basically mediate students to quit from learning, so as to avoid revealing their incapability in front of others.

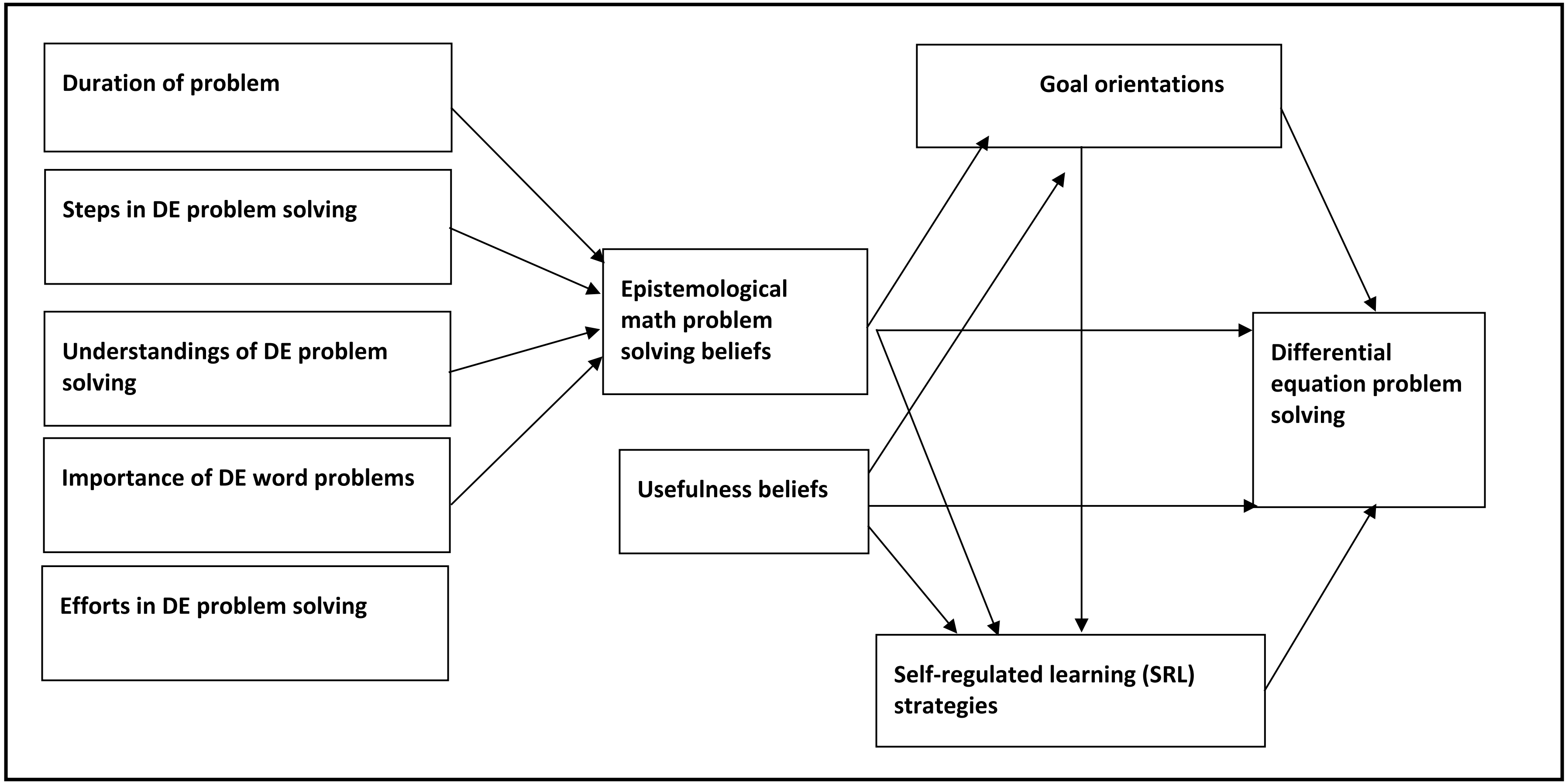

Figure 1: The proposed conceptual model

These goals can be assessed with the adapted scales of the Patterns of Adaptive Learning Survey (Midgley et al., 2000). Mastery goal comprised of six items, for example, “In differential equation class, understanding the work is more important to me than the grade I get”. The measure of performance goal included five items for instance, “Doing better than other students in differential equation class is important to me”. While, measure of avoidance goal included six items, such as, “It is very important to me that I don't look stupid in my differential equation class”.

The fourth construct “self-regulated learning (SRL) strategies” are basically self-generated beliefs, emotions, and actions that are intended and frequently adapted for the accomplishment of individual goals (Zimmerman et al., 2000). It is self-directive procedure through which students switch their mental abilities into academic skills. SRL may be measured by the scale taken from the Motivated Strategies for Learning Questionnaire (MSLQ) (P. R. Pintrich, 1991). Overall, MSLQ has 15 sub-scales, in which six subscale lie inside the motivation section and nine within the learning strategies section. Literature reveals that using two dimensions critical thinking and elaboration would be able to enhance problem solving ability. The elaboration strategy has six items such as “When reading for differential equation class, I try to make a connection with my previous knowledge”. The measure of critical thinking included five items such as “I treat the differential equation course material as a starting point and try to cultivate my own thoughts about it”.

Through investigation of direct effect of each factor individually, as well as through mediating factors (such as goal orientation and/or self-regulated learning), several outcomes may be possible. Therefore, the authors are further working in these directions.

CONCLUSIONS

In this study, different factors affecting students’ differential equations (DEs) solving abilities were explored at pre university level. It was concluded that the four factors; epistemological math problem solving beliefs, belief about usefulness of mathematics, self-regulated learning (SRL) strategies and goal orientations have great potential to enhance differential equation problem ability. In the second phase of this study, a conceptual model was developed to combine these four factors for effective differential equation problem solving. Through investigation of direct effect and mediating role of each factor, several outcomes were anticipated.

This work would further enable the development of a structural equation model (SEM) relating the four factors, including epistemological math problem solving beliefs, usefulness, self-regulated learning strategies, and goal orientation beliefs, to support the teaching and learning of differential equation problems at pre-university level students. Also, there is a need to future study to test this model in quantitative research. Therefore, authors are further testing the predictive ability of this model in differential equation problem solving. Several research hypotheses would be evaluated using Partial Least Square Structural Equation Modeling (PLS-SEM).

In addition to SEM, it is anticipated that the results of this model would provide a new avenue to educators and teachers to overcome the students’ problem by boosting up the students’ mind psychological. Findings of this conceptual model would add useful information to available literature to fill the research gap generally for curriculum implementation, and teaching and learning of differential equation effectively. Authors are further working in these directions to provide several important implications for the curriculum designers and teachers to enhance conceptual understanding in differential equation problem solving, particularly in the developing countries.

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