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DESIGN AND OPTIMIZATION OF LADDER WAVE DIGITAL FILTERS

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ABSTRACT

This paper introduces a time domain approach for designing ladder wave digital filters (WDFs) with limited coefficient wordlengths. Simple iteration equations yield the time domain response of a ladder WDF. An optimization algorithm then facilitates the design of the WDF to meet a desired response with a specific number of bits representing the coefficients. This approach also affords realizations with frequency domain constraints.

Keywords: Optimization, Wave digital filters

1.0 INTRODUCTION

The cost of implementing digital filters as hardware depends heavily on the wordlength of the coefficients. It is therefore imperative that the wordlength is designed to be as short as possible. One method of ensuring this is to select a digital filter that is inherently insensitive to variations of its coefficient values. Wave digital filters (WDFs) are known to exhibit this highly desirable property [1].

WDFs are based on analogue filters with resistive terminations as models. There are many types of analogue filters. The major types that have been used as models to wave digital filters are the ladder, lattice and unit-element analogue filters. Ladder analogue filters translate to ladder WDFs. These analogue filters consist of inductors and capacitors arranged in the ladder configuration and are resistively terminated. These reference analogue filters are inherently insensitive to their element value variations [2]. WDFs emulate the structures of the analogue filters and carry over the sensitivity properties through to the digital domain. The frequency and impulse responses of the WDFs are insensitive to small variation in the multiplier coefficient values. This translates to realizations with relatively short coefficient wordlengths in comparison with other types of digital filters. The coefficient wordlengths are found by using the simple rounding technique [7].

Further reduction of the wordlength can be achieved by optimizing the rounded filter coefficients in the discrete parameter space. Although the rounding technique results in WDFs with relatively short coefficient wordlengths, the response of the WDF may exceed given specifications at some time and frequency intervals. This paper demonstrates a time domain approach for optimizing the WDF coefficient wordlength to meet a specified response.

WDFs have complex structures in comparison with other types of filters. However, by rearranging and dividing the equations of the ladder WDFs into two distinct parts, simple iteration equations can be used to generate the impulse responses. The time domain responses are combined with the established pattern search optimization algorithm [3] to reduce the coefficient wordlength to a minimum number of bits. By invoking Parseval's theorem [4], a frequency domain specification can be met using this time domain approach.

2.0 LADDER WDFS

2.1 Structure

There are many types of WDFs reflecting the many varieties of analogue filters from which they are derived. Fig. 1 shows a reference analogue ladder filter.

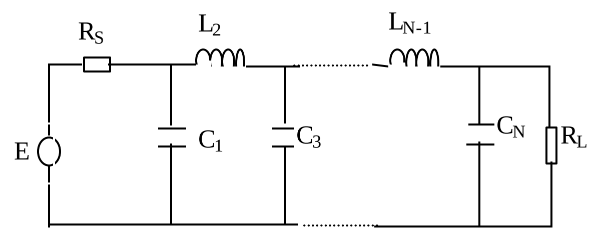

Fig. 1: Analogue ladder filter

Ladder WDFs mimic the structure of ladder analogue filters. These WDFs consist of a cascade of three-port adaptors that house the digital adders and multipliers. The adaptors simulate the series and parallel connections of the inductors and capacitors of the analogue reference filters. In theory it is possible to realize ladder WDFs by directly interconnecting the three-port adaptors [5]. However, in practice, implementation is impossible since this requires simultaneous calculations of all adaptor equations, and the results are required by the adjacent adaptors. A more practical approach to implementation is to place delay elements in between the adaptors [6]. This results in a configuration that can be realized with a valid computational sequence. This requires the modification of the reference ladder filters, where unit elements (UEs) are inserted and shifted to the appropriate places using Kuroda's transformation [7].

Fig. 2a shows the modified analogue ladder filter while Fig. 2b shows the corresponding configuration of a ladder WDF realized with three-port parallel adaptors. The translation from the analogue domain, Fig. 2a, to the digital domain, Fig. 2b, requires the use of a bilinear transformation [1]. This results in the translation of each analogue components to its digital equivalent. The adaptors are then required to interconnect the digital elements. In this case, the parallel adaptors are used to connect the elements in parallel.

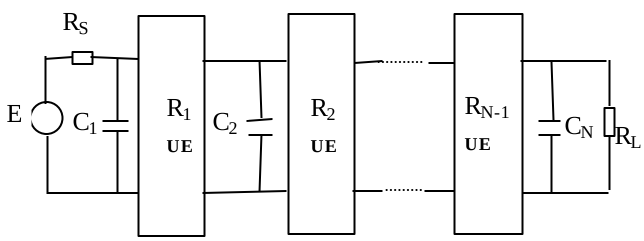

Fig. 2a: Modified analogue ladder filter

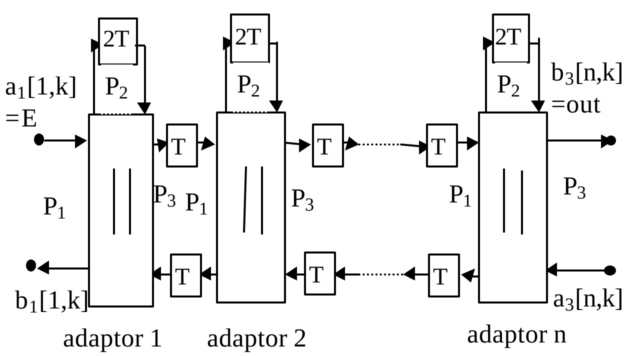

Fig. 2b: Corresponding ladder WDF with parallel adaptors

Ladder WDFs can be realized with three-port series adaptors by suitably modifying the analogue filters. An alternative method of using Kuroda's transformation is to eliminate the capasitive components of Fig. 1. Fig. 3a shows the modifications required while Fig. 3b shows the corresponding ladder WDF. In Fig. 3b, the series adaptors are used to represent the series connections of the digital components, reflecting the series connections of its analogue reference filters.

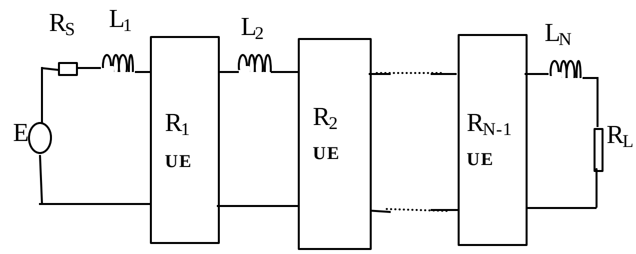

Fig. 3a: Modified analogue ladder filter

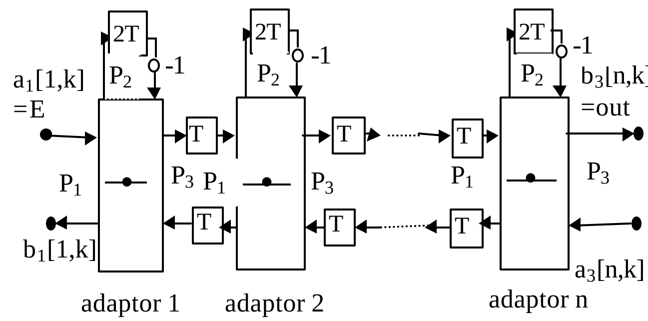

Fig 3b: Ladder WDF with series adaptors

2.2 Adaptor Equations

The equations that describe ladder WDFs can be divided into two parts. The first kind describes the adaptor functions. For the three-port parallel adaptor, with *a* as the input signal, *b* the output signal, *g* the multiplying coefficient and *p* indicating the port number, the equation of the adaptor is [1].

$$b_p = -a_p + g_1 a_1 + g_2 a_2 + g_3 a_3, \quad p = 1..3 \tag{1}$$

The number of multipliers can be reduced from three to two by using the principle of port dependence. If the third port is selected as the dependent port, the equations of the three-port parallel adaptor are:

$$b_3 = a_3 - g_1(a_3 - a_1) - g_2(a_3 - a_2) \tag{2.a}$$

$$b_p = b_3 + (a_3 - a_p), \quad p = 1, 2 \tag{2.b}$$

For the series adaptor with no dependent port, the equation that describes it is:

$$b_p = a_p - g_p(a_1 + a_2 + a_3), \quad p = 1..3 \tag{3}$$

If port three is selected as the dependent port, the number of multipliers per adaptor reduces from three to two. The equations of the adaptor are:

$$b_3 = -(b_1 + b_2 + a_1 + a_2 + a_3) \tag{4.a}$$

$$b_p = a_p - g_p(a_1 + a_2 + a_3), \quad p = 1, 2 \tag{4.b}$$

2.3 Connection Equations

The second kind of WDF equations describes the interconnections of the adaptors. Denoting n as the adaptor number and k the iteration number, for the parallel adaptors the connecting equations are:

$$a_1[n,k+1] = b_3[n-1,k] \quad (5.a)$$

$$a_2[n,k+2] = b_2[n,k] \quad (5.b)$$

$$a_3[n,k+1] = b_1[n+1,k] \quad (5.c)$$

And for the series adaptors, these are:

$$a_1[n,k+1] = b_3[n-1,k] \quad (6.a)$$

$$a_2[n,k+2] = -b_2[n,k] \quad (6.b)$$

$$a_3[n,k+1] = b_1[n+1,k] \quad (6.c)$$

The adaptor and connection equations can be used to find the impulse response of a ladder WDF.

3.0 TIME DOMAIN OPTIMIZATION

The direct search method of Hooke and Jeeves [3] has been used for the finite wordlength design of digital filters in the frequency domain [4]. Here it is applied in the time domain for WDFs. Modifications [8] to the original algorithm resulting in simultaneously changing more than one coefficient when the objective function is not minimized are included here. This method is called the direct search method because it is used to describe the sequential examination of trial solutions. This search is also called the pattern search because it determines the sequence of values for the argument.

The basic strategy of the main program is as the following: the current value of the parameter vector is ψ, and φ is the exploratory value. A subroutine performs a univariate search by incrementing and decrementing each component of φ, accepting any improvements in the function F. If an improvement is found, θ is set to ψ, saving the old value; ψ is set to φ, updating the current value; and φ is set to $2\varphi-\theta$, thereby attempting to extrapolate the pattern of the successful move. The subroutine is then invoked for a local exploration about φ. The process is continued until no improvement is found. If at least one improvement is found during this process, the strategy is applied again in an attempt to build up a new pattern. If the pattern move fails to reduce the value of the function, the point from which it starts is retained.

After the pattern move is tried, the whole process is repeated, starting with a new local exploratory search, followed by a pattern move, followed by a local exploratory search, and so on. When a local exploratory fails, the last point determined by the algorithm is a minimum of the function. When finally no improvement is found in this attempt, the step size ∇ is reduced by a factor of 2 and the process repeated. The algorithm terminates when ∇ falls below some prescribed value. First one directional search is considered. After changing the ∇, if there is no improvement, another subroutine [8] is used to change two parameter values at the same time. In this way, stronger local optima are found.

Time domain optimization of a ladder wave digital filter begins with an initial impulse response with the coefficients having an infinite precision. The objective function to be minimized is:

$$E(\bar{g}) = \max\{\frac{1}{d^2(t_k)}\left|H(t_k) - H^*(t_k)\right|^2\} \quad (7)$$

where $\bar{g}$ is the set of filter coefficients, $d(t_k)$ the tolerance function, $H(t_k)$ the desired magnitude, $H^*(t_k)$ the calculated magnitude at the time t_k. This error is calculated at a fixed grid where the intervals are dictated by the sampling period of the ladder WDFs.

4.0 FREQUENCY DOMAIN EQUIVALENCE

Specifications that are given in the frequency domain can be met in the time domain by invoking Parseval's theorem. The orthogonality of the basis functions of the Fast Fourier Transform (FFT) ensures the validity of this approach. The objective function for optimization on the frequency grid is

$$E(\bar{g}) = \max\{\frac{1}{2pd^2(f_k)}\left|H(f_k) - H^*(f_k)\right|^2\} \quad (8)$$

where d, H and H^* are as defined previously but evaluated at frequency f_k.

The difference between equations (7) and (8) is the constant factor 2π. By suitably adjusting the tolerance factor, frequency domain specifications can be met. It should be noted, however, that no distinction can be made between the tolerance in the passband and stopband using this approach. The stricter tolerance should be selected for optimization to ensure that specifications are not violated at any frequency. Choosing a very strict tolerance may however result in a wordlength that is greater than the desired wordlength. A trade-off between the tolerance and the smallest achievable coefficient wordlength is inherent in this optimization approach. However, this technique can still be used to reduce the coefficient wordlength to the shortest possible value that still meets the tolerance specifications.

5.0 EXAMPLE

As an example, a 5th order ladder WDF is optimized in the time domain. The wordlength of the coefficients is selected as 6 bits with one bit representing the sign. The ladder WDF consists of 5 three-port series adaptors. The infinite precision multiplying co-efficients are shown in Table 1a and 1b. The values of the decimal equivalent of the coefficients rounded and optimized to 6 bits are shown in Tables 2 and 3 respectively.

Table 1a: Multiplying coefficient values of the example ladder WDF for the first 3 adaptors

	Adaptor 1	Adaptor 2	Adaptor 3
γ_1	0.8062891	0.1287144	0.8252915
γ_2	0.1959282	0.8172043	0.06579652

Table 1b: Multiplying coefficient values of the example ladder WDF for the last 2 adaptors

	Adaptor 4	Adaptor 5
γ_1	0.03330992	0.1669261
γ_2	0.05050596	0.8950557

Table 2: Multiplying coefficient values rounded to 6 bits

	Adaptor 1	Adaptor 2	Adaptor 3	Adaptor 4	Adaptor 5
γ_1	0.812500	0.125000	0.812500	0.031250	0.156250
γ_2	0.187500	0.812500	0.062500	0.062500	0.906250

Table 3: Multiplying coefficient values optimized to 6 bits

	Adaptor 1	Adaptor 2	Adaptor 3	Adaptor 4	Adaptor 5
γ_1	0.843750	0.125000	0.812500	0.031250	0.156250
γ_2	0.187500	0.750000	0.062500	0.062500	0.781250

The design procedures for ladder wave digital filters, from specifications to the required filter order and coefficient values are detailed by [9].

Fig. 4 shows the infinite precision response corresponding to the ideal response, the response obtained by rounding the coefficients to 6 bits, and the response obtained by optimizing to 6 bits. The figure clearly indicates the superiority of the optimized response. The target tolerance for all time samples is selected as 0.016. The program is written in the C language. It requires 1047 function evaluations and takes 35 seconds for convergence on a personal computer with a 75 MHz pentium microprocessor.

Fig. 5 shows the frequency response curves obtained by taking the Fast Fourier Transform of the impulse responses. The filter has a cut-off frequency of 400 Hz and a sampling frequency of 10 KHz. The difference between the response curve with infinite precision coefficients and that optimized to 6 bits does not exceed 0.1 in accordance to equation (8).

In fact, the difference obtained for this example does not exceed 0.03. This is found to be typical using this approach where the actual tolerance exceeds the estimated tolerance in the frequency domain. For this example, 6-bits is the minimum wordlength obtainable that meets the specifications.

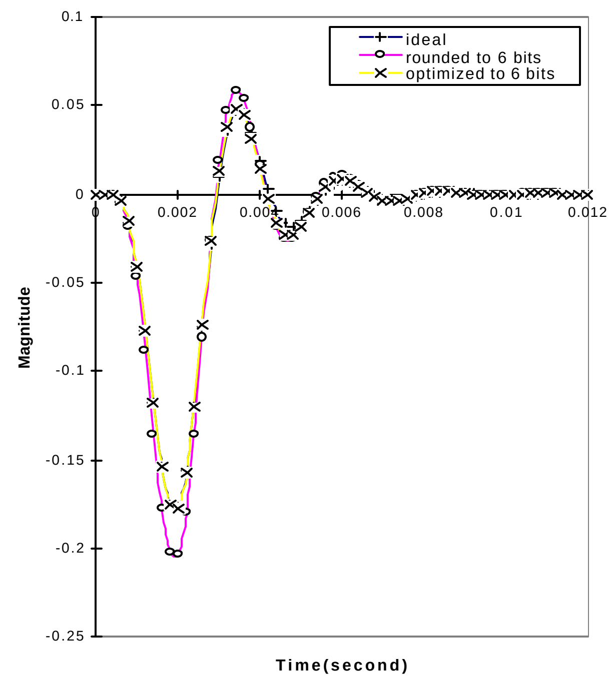

Fig. 4: The impulse responses of a 5th order ladder WDF showing the ideal response, the response with the coefficients rounded to 6 bits and the response with the coefficients optimized to 6 bits

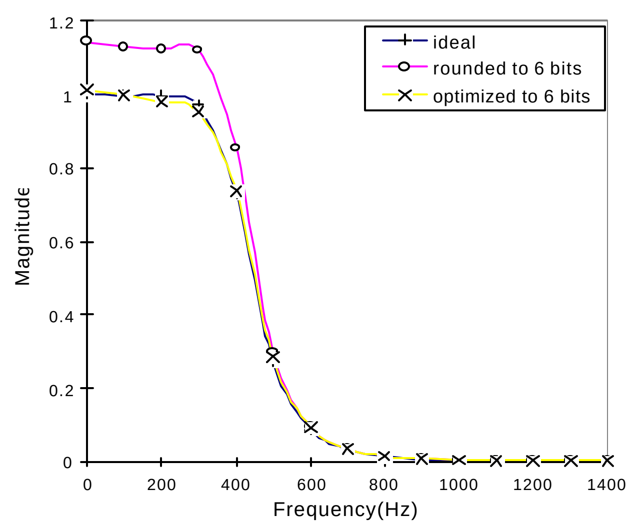

Fig 5: The corresponding frequency responses of Fig. 4

6.0 CONCLUSIONS

This paper has demonstrated a time domain approach for designing ladder WDFs with limited coefficient wordlengths. The time domain responses of ladder WDFs are generated using equations that describe the adaptors and the interconnection between these adaptors. The optimization algorithm ensures that the time domain response for a specific wordlength does not exceed a given tolerance. Correspondence with the frequency response is shown for design flexibility. This algorithm stresses on the use of simple iteration equations to obtain the impulse responses of the ladder WDFs. This combined with the fast converging optimization algorithm can be used to design a ladder wave digital filter with the minimum coefficient wordlength that meets the required specifications.

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